Book of Abstracts

Symmetry and shape

13 - 16 October 2022

Santiago de Compostela, Spain

http://xtsunxet.usc.es/symmetry2022/

Welcome

According to Felix Klein, geometry is the study of those properties in space that are invariant under a given transformation group. Intuitively, symmetry is the correspondence of shape at every point of a space. An interesting problem in geometry and many physical sciences is to determine the symmetries of a space from its shape.

The aim of this conference is to gather experts in the study of symmetry in Differential Geometry. The conference will revolve around the study of curvature, homogeneous and symmetric spaces, Riemannian submanifold geometry, and other related topics in Differential Geometry and Geometric Analysis.

Contents

Organization

Organizing committee

José Carlos Díaz Ramos, Universidade de Santiago de Compostela, Spain
Miguel Domínguez Vázquez, Universidade de Santiago de Compostela, Spain
Eduardo García Río, Universidade de Santiago de Compostela, Spain
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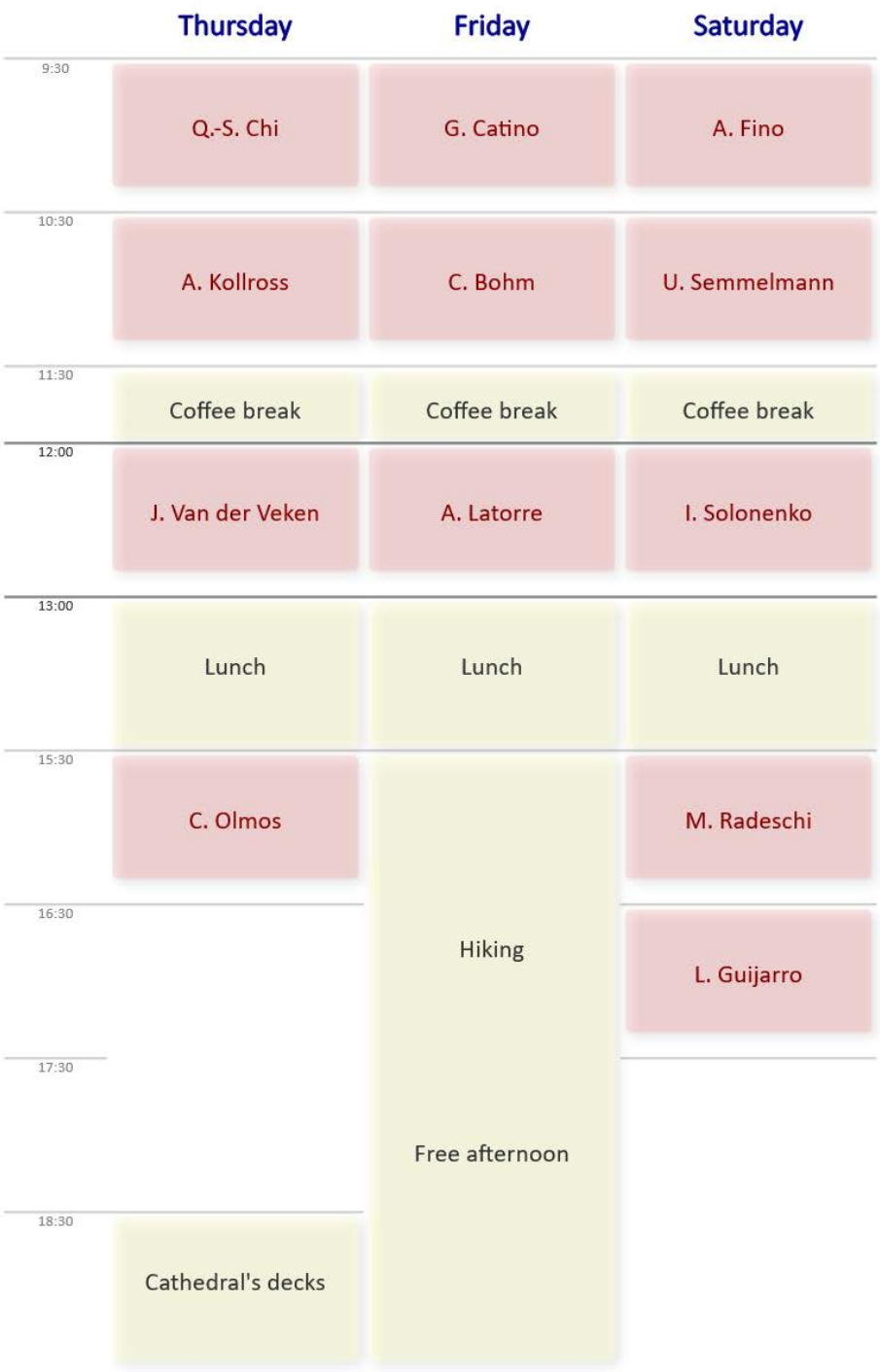
Schedule

The scientific programme is composed of invited lectures. All lectures will take place in Aula Magna of the Faculty of Mathematics.



Practical information

Internet connection

You can connect via **eduroam**, if your affiliation supports it and you have your devices correctly configured.

Restaurants

There are several options to have lunch near the conference venue:

- Some restaurants near the campus, such as *Santos*, *Xugo*, *Xantar* or *Altamira*, offer lunch menus for around 10-12 euros.
- Downtown and in the historic center there are many kinds of restaurants. Vegan and vegetarian options can be found, for example, at *Boca a boca*, *Malak Bistro*, or *The Veggie Carmen*. Many restaurants offer typical Galician food, such as *O Dezaseis* or *María Castaña*.
- The cafeteria-canteens of the Faculty of Mathematics (same building as the conference), of the Escola Técnica Superior de Enxeñaría - ETSE (200m), or the cafeterias Fonseca and Rodríguez-Cadarso (300m) offer menus for 6 euro. Because of students' schedule, it is recommended to have lunch before 13:30.

Abstracts

Invited talks

The Alekseevskii conjecture

Christoph Böhm
(Universität Münster, Germany)

We will report on recent progress towards the classification of homogeneous Einstein metrics. While homogeneous Ricci-flat spaces are flat, homogeneous Einstein spaces with negative Einstein constant are diffeomorphic to a Euclidean space, as predicted by the Alekseevski conjecture. If time permits we will indicate some ideas of the proof. The classification of homogeneous Einstein spaces with positive Einstein constant is wide open, even though there are general existence and non-existence results.

Some canonical metrics on Riemannian manifolds

Giovanni Catino
(Politecnico di Milano, Italy)

In this talk I will review recent results concerning the existence of some canonical Riemannian metrics on closed (compact with no boundary) smooth manifolds. The constructions of these metrics are based on Aubin's local deformations and a variant of the Yamabe problem which was first studied by Gursky.

Fano 3-folds and classification of constantly curved holomorphic 2-spheres of degree 6 in the complex Grassmannian $G(2,5)$

Quo-Shin Chi
(Washington University at St. Louis, USA)

Harmonic maps from the Riemann sphere to Grassmannians (and, more generally, to symmetric spaces) arise in the sigma-model theory in Physics. Such maps of constant curvature constitute a prototypical class of interest, for which Delisle, Hussin and Zakrzewski proposed the conjecture that the maximal degree of constantly curved holomorphic 2-spheres in the (complex) $G(m,n)$ is $m(n-m)$ and confirmed it when $m=2$ and $n=4$ or 5. Up to unitary equivalence, there is only one constantly curved holomorphic 2-sphere of maximal degree 4 in $G(2,4)$ by Jin and Yu. On the other hand, up to now, the only known example in the literature of constantly curved holomorphic 2-sphere of maximal degree 6 in $G(2,5)$ has been the first associated curve of the Veronese curve of degree 4. By exploring the rich interplay between the Riemann sphere and projectively

equivalent Fano 3-folds of index 2 and degree 5, we prove, up to the ambient unitary equivalence, that the moduli space of (precisely defined) generic such 2-spheres is semi-algebraic of dimension 2. All these 2-spheres are verified to have non-parallel second fundamental form except for the above known example.

Special Hermitian metrics and suspensions

Anna Fino
(University of Torino, Italy)

In the talk I will report some general results on existence of special hermitian structures, like balanced, SKT and generalized Kähler, on suspensions.

In particular, I will show some recent results on compact solvmanifolds, in collaboration with Fabio Paradiso, and on toric suspensions of hyperKähler manifolds, in collaboration with Gueo Grantcharov and Misha Verbitsky.

Some topological and geometrical consequences of positive intermediate Ricci curvature

Luis Guijarro
(Universidad Autónoma de Madrid, Spain)

Intermediate Ricci curvature appears naturally when taking traces of the transversal Jacobi equation. We will show how conditions on its sign restrict the geometry and the topology of a manifold in two different situations: first, we will examine the interplay between the second fundamental form of submanifolds and its focal radius; second, we will use this to provide versions of Wilking's Connectivity Theorem and of Fraenkel's Theorem on the intersection of totally geodesic submanifolds.

This is joint work with Fred Wilhelm (UCR).

Polar actions on Damek-Ricci spaces

Andreas Kollross
(Universität Stuttgart, Germany)

A proper isometric Lie group action on a Riemannian manifold is called polar if there exists a closed connected submanifold which meets all orbits orthogonally. Most known examples of polar actions are related to symmetric spaces. In this talk, after a short survey on classification results, I will talk about polar actions on Damek-Ricci spaces. Using an earlier result of S. Kim, Y. Nikolayevsky and J. Park on totally geodesic subspaces of Damek-Ricci spaces, I will give examples and present some partial classifications of polar actions on Damek-Ricci spaces. In particular, I will show that non-trivial polar actions exist on all Damek-Ricci spaces.

Behaviour of pseudo-Kähler structures under holomorphic deformations

Adela Latorre
(Universidad Politécnica de Madrid, Spain)

Let M be a $2n$-dimensional differentiable manifold. A pseudo-Kähler structure on M is a pair (J, ω), where J is a complex structure and ω is a symplectic form that satisfy the compatibility condition

$$\omega(J\cdot, J\cdot) = \omega(\cdot, \cdot).$$

When $g(\cdot, \cdot) = \omega(\cdot, J\cdot)$ is a positive definite metric, the manifold (M, J, ω) is called Kähler. Then, a well-known result of Kodaira and Spencer states that for every small deformation J_t of the initial complex structure $J_0 := J$ one can find a symplectic form ω_t that makes (M, J_t, ω_t) a Kähler manifold for every sufficiently small $t \neq 0$.

In this talk, we will see that a similar behaviour cannot be guaranteed for pseudo-Kähler manifolds, namely, when the initial metric g is not positive definite. This will lead us to address the problem of finding conditions on the families of complex manifolds (M, J_t) under which the existence of ω_t compatible with J_t is preserved. Time permitting, we will also analyze this problem for other related structures, such as neutral Calabi-Yau metrics.

Homogeneous Riemannian manifolds with nullity

Carlos Olmos
(Universidad Nacional de Córdoba, Argentina)

We will speak about joint results with *Antonio J. Di Scala* and *Francisco Vittone*, about the structure of *irreducible* homogeneous Riemannian manifolds $M^n = G/H$ whose curvature tensor has non-trivial nullity. In a recent paper we developed a general theory to deal with such spaces. By making use of this theory we were able to construct the first non-trivial examples in any dimension. Such examples have the minimum possible conullity $k = 3$. The key fact is the existence of a non-trivial transvection X at p (i.e. Killing fields $(\nabla X)_p = 0$) such that X_p is not in the nullity subspace ν_p at p but the Jacobi operator $R_{\cdot, X_p} X_p$ is zero. The nullity distribution ν is highly non-homogeneous in the sense that no non-trivial Killing field lie in ν and hence ν is not given by the tangent spaces of orbits of an isometry subgroup of G. The Lie algebra $\mathfrak{g}$ of G is never reductive (in particular, M is not compact).

After surveying on these results we will present recent results that give a substantial improvement for the structure of homogeneous spaces in relation with the nullity. By some rather delicate argument we showed that there is always a transvection, possibly enlarging the presentation group G, in any direction ν_p of the nullity, for all $p \in M$. Moreover, such transvections form an abelian ideal $\mathfrak{a}$ of $\mathfrak{g}$ which implies, if $k = 3$, that $G = \mathbb{R}^{n-1} \rtimes \mathbb{R}$ and H is trivial.

On the one hand, the leaves of the nullity are Euclidean spaces and the projection to the quotient space M/ν is never a Riemannian submersion. But on the other hand, the foliation given by the orbits of the normal subgroup $A \subset G$, associated to $\mathfrak{a}$ is intrinsically flat, contains the nullity foliation and the projection to the quotient is a Riemannian submersion.

Finally, we will show simply connected examples with non-trivial topology and compact quotients. This answers a natural question.

When is an orbifold, a manifold?

Marco Radeschi
(University of Notre Dame, USA)

Riemannian orbifolds are metric spaces locally isometric to a quotient of a Riemannian manifold, by a finite group of isometries. For such spaces, one can define orbifold homotopy and (co)-homology groups that, unlike their standard counterparts, contain information about the local quotients. In this talk, I survey a few recent results about finding geometric and topological conditions on an orbifold, which ensure it is in fact a manifold. In particular, I will talk about a recent joint work with Christian Lange, proving that if an n-orbifold is $n/2$-connected in the orbifold topology, then it is a manifold.

Totally geodesic submanifolds in Hopf-Berger spheres

Alberto Rodríguez-Vázquez
(Universidade de Santiago de Compostela, Spain)

The classification of transitive Lie group actions on spheres was obtained by Borel, Montgomery, and Samelson in the forties. As a consequence of this, it turns out that apart from the round metric there are other Riemannian metrics in spheres which are invariant under the action of a transitive Lie group. These other homogeneous metrics in spheres can be constructed by modifying the metric of the total space of the complex, quaternionic or octonionic Hopf fibration in the direction of the fibers.

In this talk, I will report on an ongoing joint work with Carlos Olmos (Universidad Nacional de Córdoba), where we classify totally geodesic submanifolds in Hopf-Berger spheres. These are those Riemannian homogeneous spheres obtained by rescaling the round metric of the total space of Hopf fibrations by a positive factor in the direction of the fibers.

Quaternion Kähler manifolds of non-negative curvature

Uwe Semmelmann
(Universität Stuttgart, Germany)

In my talk I will discuss a (still unproved) conjecture for quaternion Kähler manifolds. For this I will present a new formulation of the proof for the analogous statement on Kähler manifolds, originally due to A. Gray. I will explain why a proof of the conjecture given by Chow and Yang is not correct and will make a few additional comments on the curvature of Wolf spaces. My talk is based on discussions with Gregor Weingart and Oscar Macia.

Lagrangian submanifolds of the complex (hyperbolic) quadric

Joeri Van der Veken
(University of Leuven, Belgium)

In this talk we discuss submanifolds of the complex quadric and the complex hyperbolic quadric. Both are Kähler-Einstein spaces with a particular geometric structure: they carry a family of non-integrable almost product structures which anti-commute with the complex structure. Most of the talk will be about Lagrangian submanifolds of these spaces, which can be seen as images of Gauss maps of hypersurfaces of spheres and of spacelike hypersurfaces of anti-de Sitter spaces respectively.

Posters

Local topological rigidity of 3-manifolds of hyperbolic type

Andrea Drago
(Sapienza University of Rome, Italy)

We study systolic inequalities for closed, orientable, Riemannian 3-manifolds of bounded positive volume entropy. This allows us to prove that the class of atoroidal manifolds (i.e. that admit an hyperbolic metric) with uniformly bounded diameter and volume entropy is topologically rigid. In particular our *main result* is the following theorem:

Let X be a closed, orientable, atoroidal, Riemannian 3-manifold with $\mathrm{Ent}(X) < E$ and $\mathrm{Diam}(X) < D$. Then there exist a function $s(E, D)$ such that, if Y is closed, orientable, torsionless, Riemannian 3-manifold with $\mathrm{Ent}(Y) < E$ and $d_{GH}(X, Y) < s(E, D)$, then $\pi_1(X) \cong \pi_1(Y)$. In particular, X and Y are diffeomorphic.

Homogeneous spaces of G_2

Cristina Draper Fontanals
(Universidad de Málaga, Spain)

Pilar Benito, Cristina Draper and Alberto Elduque study the reductive homogeneous spaces obtained as quotients of the exceptional group G_2 in the Draper doctoral dissertation (see [1]), from an algebraic perspective. In this poster we revisit these spaces from a more geometrical approach.

References:

[1] P. Benito, C. Draper, A. Elduque: Lie-Yamaguti algebras related to $\mathfrak{g}_2$, *J. Pure Appl. Algebra* **202** (2005), 22-54.

Codimension two polar homogeneous foliations on noncompact symmetric spaces

Juan Manuel Lorenzo-Naveiro
(Universidade de Santiago de Compostela, Spain)

An isometric action of a Lie group on a Riemannian manifold is polar if there exists a complete submanifold (called a section) that intersects every orbit perpendicularly. It is an open problem to determine all such actions up to orbit equivalence on ambient manifolds with a large group of isometries (in particular, symmetric spaces).

We will focus on polar actions without singular orbits on symmetric spaces of noncompact type. In this setting, a complete classification of cohomogeneity one actions is known [2], while in codimension two there is a procedure to construct all possible foliations in which the section is the Euclidean plane, derived from a more general method described in [1]. We will show how to finish the case of codimension two by determining all polar foliations whose section is homothetic to the hyperbolic plane.

References:

[1] J. Berndt, J. C. Díaz-Ramos, H. Tamaru: Hyperpolar homogeneous foliations on symmetric spaces of noncompact type, *J. Differential Geom.* **86** (2012), no. 2, 191-235.
[2] J. Berndt, H. Tamaru: Homogeneous codimension one foliations on noncompact symmetric spaces, *J. Differential Geom.* **63** (2003), no. 1, 1-40.

Rigidity of weighted Einstein manifolds

Diego Mojón Álvarez
(Universidade de Santiago de Compostela, Spain)

A smooth metric measure space (SMMS) is a 5-tuple (M^n, g, f, m, μ), where (M, g) is a Riemannian manifold, $f \in C^\infty(M)$ is a density function, $m \in \mathbb{R}^+$ is a dimensional parameter, and $\mu \in \mathbb{R}$ is an auxiliary curvature parameter. The study of the geometry of SMMS relies on weighted objects, which retain certain geometric properties while incorporating information about the density function.

We consider the weighted analogues of tensors associated to curvature. Under the weighted Einstein condition, we obtain some rigidity results when the weighted Weyl tensor is weighted harmonic, showing that the underlying structure of the manifold is that of a warped product with 1-dimensional base and Einstein fiber. Moreover, if the scalar curvature is constant then the manifold is Einstein in the usual sense.

Homogeneous Hypersurfaces on Symmetric Spaces

Tomás Otero-Casal
(Universidade de Santiago de Compostela, Spain)

Riemannian symmetric spaces provide a particularly nice setting in which to study (extrinsically) homogeneous submanifolds, which arise as orbits of isometric actions. This poster reports on some recent developments on the classification of homogeneous hypersurfaces in symmetric spaces of noncompact type where, in a joint work with J. Carlos Díaz-Ramos and Miguel Domínguez-Vázquez [1] we have proposed a new structural result about cohomogeneity one actions.

References:

[1] J. C. Díaz-Ramos, M. Domínguez-Vázquez, T. Otero: Cohomogeneity one actions on symmetric spaces of noncompact type and higher rank. arXiv:2202.05138.

The κ-nullity of Riemannian manifolds and their splitting tensors

Felippe Soares Guimarães
(IME-USP / KU Leuven, Brazil / Belgium)

We consider Riemannian n-manifolds M with nontrivial κ-nullity "distribution" of the curvature tensor R, namely, the variable rank distribution of tangent subspaces to M where R coincides with the curvature tensor of a space of constant curvature κ ($\kappa \in \mathbb{R}$) is nontrivial. We obtain classification theorems under different additional assumptions, in terms of low nullity/conullity, controlled scalar curvature or existence of quotients of finite volume. We prove new results, but also revisit previous ones.

On a class of system of partial differential equations describing pseudo-spherical or spherical surfaces

Filipe Kelmer
(University of Brasília, Brazil)

We consider systems of partial differential equations of the form

$$\begin{cases} u_{xt} = F\left(u, u_x, v, v_x\right), \\ v_{xt} = G\left(u, u_x, v, v_x\right), \end{cases}$$

describing pseudospherical (**pss**) or spherical surfaces (**ss**), meaning that, their generic solutions $(u(x,t), v(x,t))$ provide metrics, with coordinates (x,t), on open subsets of the plane, with constant curvature $K=-1$ or $K=1$. These systems can be described as the integrability conditions of $\mathfrak{g}$-valued linear problems, with $\mathfrak{g}=\mathfrak{sl}(2,\mathbb{R})$ or $\mathfrak{g}=\mathfrak{su}(2)$, when $K=-1$, $K=1$, respectively. We obtain characterization and also classification results. Applications of the theory provide new examples and new families of systems of differential equations, which also contain generalizations of a Pohlmeyer-Lund-Regge type system and of the Konno-Oono coupled dispersionless system. We provide explicitly the first few conservation laws, from an infinite sequence, for some of the systems describing **pss**.

References:

[1] F. Kelmer, K. Tenenblat: On a class of systems of hyperbolic equations describing pseudo-spherical or spherical surfaces, *J. Differential Equations* **339** (2022), 372-394.

Participants

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