A CLASSIFICATION FOR ALMOST CONTACT STRUCTURES, II: EXAMPLES

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INTRODUCTION

In the previous paper [7] we gave a classification for almost contact metric (a. ct. m.) structures on odd-dimensional manifolds. Doing this, some new classes of a. ct. m. structures have been defined and all the inclusion relations between old and new classes have been stablished.

The main point of the present paper is to study whether the inclusion relations in our classification are strict; this is done by the method of constructing explicit examples. In fact, these examples show the strictness of the inclusions for all the cases except two, one of which is actually an equality and the other remains as an open question.

The paper is structured in three paragraphs. §1 and §2 are devoted to the framework of semi-cosymplectic and semi-Sasakian structures respectively, and §3 is devoted to the remaining classes.

The technique we use to construct each appropriate example goes as follows: we consider an almost-Hermitian manifold (M,J,h) and make the product of M with $\mathbb{R}$ or with an odd-dimensional unit sphere; then, a suitable a. ct. m. structure can be defined on the product manifold and this a. ct. m. structure provides the examples, depending on the class of the almost Hermitian structure considered on M and on the metric on the product manifold. Of course, many of the examples constructed here belong to the new classes and, thus, the non triviality of them is demonstrated. In particular, we must emphasize the case of the called trans-Sasakian structures; this class plays a central role in our classification because it contains and, in a certain sense, separates both cosymplectic and Sasakian structures, such as the quasi-Sasakian class (Blair [1]) does. Here we show, by constructing explicit examples, that no inclusion relation exists between both trans-Sasakian and quasi-Sasakian structures; actually, we give examples of manifolds of each of these classes which do not belong to the other.

Notations and terminology in this paper will be the same as those employed in [7], to which we refer for the explicit definitions of the different classes of a. ct. m. structures here involved. Furthermore, when speaking of an a. ct. m. manifold, cosymplectic manifold, Sasakian manifold, etc., we shall mean the manifold with the corresponding structure and $|C|$, $|S|$, etc., will denote respectively the class of cosymplectic manifolds, Sasakian manifolds, etc.

I wish to express here my hearty thanks to Prof. L.A. Cordero for his valuable suggestions and guidance.

1. SEMI-COSYMPLECTIC STRUCTURES

In this section we shall be concerned with those classes of a. ct. m. structures which can be assembled under the common denomination of semi-cosymplectic structures; the inclusion relations between them, as stablished in [7], are graphically shown in the following diagram

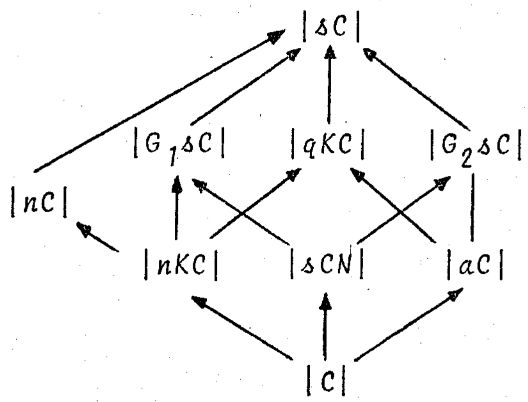

DIAGRAM I

where the arrows (⟶) mean inclusions ($\subset$).

In order to construct examples of manifolds with semi-cosymplectic structures, which make the strictness of the inclusions in Diagram I clear, we shall proceed as follows.

Let M be an almost Hermitian manifold, dim $M = 2n$, with metric h, almost complex structure J, Riemannian connection D and Kaehler form F defined by $F(X,Y) = h(X,JY)$, $X,Y \in \mathfrak{X}(M)$, the Lie algebra of C^∞ vector fields on M. Next, let us consider the product manifold $M\times\mathbb{R}$; a vector field on $M\times\mathbb{R}$ will be denoted by $(X,a\frac{d}{dt})$, where $X \in \mathfrak{X}(M)$, t is the coordinate of $\mathbb{R}$ and a is a C^∞ function on $M\times\mathbb{R}$. As it is well known, starting from the almost Hermitian structure (J,h) on M, an a. ct. m. structure can be defined on $M\times\mathbb{R}$ by setting

$$\varphi(X,a\tfrac{d}{dt}) = (JX,0), \qquad \xi = (0,\tfrac{d}{dt}), \qquad \eta(X,a\tfrac{d}{dt}) = a, \tag{1.1}$$

$$g((X,a\tfrac{d}{dt}),(Y,b\tfrac{d}{dt})) = h(X,Y) + ab,$$

and whose fundamental 2-form satisfies

$$\Phi((X,a\tfrac{d}{dt}),(Y,b\tfrac{d}{dt})) = F(X,Y)$$

If ∇ denotes the Riemannian connection of the metric g on $M\times\mathbb{R}$, the following identities are easily verified:

$$(\nabla_{(X,a\frac{d}{dt})}\varphi)(Y,b\tfrac{d}{dt}) = ((D_XJ)Y,0),$$

$$(\nabla_{(X,a\frac{d}{dt})}\Phi)((Y,b\tfrac{d}{dt}),(Z,c\tfrac{d}{dt})) = (D_XF)(Y,Z),$$

$$d\Phi((X,a\tfrac{d}{dt}),(Y,b\tfrac{d}{dt}),(Z,c\tfrac{d}{dt})) = dF(X,Y,Z).$$

$$\delta\Phi(X,a\tfrac{d}{dt}) = \delta F(X),$$

$$\nabla_{(X,a\frac{d}{dt})}\xi = 0, \quad \nabla_{(X,a\frac{d}{dt})}\eta = 0, \quad d\eta = 0, \quad \delta\eta = 0,$$

for arbitrary $X,Y,Z \in \chi(M)$, a,b,c C^∞ functions on $M\times\mathbb{R}$ and where δ denotes indistinctly both the coderivatives of (M,h) and of $(M\times\mathbb{R},g)$.

PROPOSITION 1.1. *We have*

(*i*) $M\times\mathbb{R} \in |C|$ (*or* $M\times\mathbb{R}$ $|tS|$) *iff* M *is Kaehlerian.*

(*ii*) $M\times\mathbb{R} \in |nKC|$ (*or* $M\times\mathbb{R}$ $|ntS|$) *iff* M *is nearly Kaehlerian.*

(*iii*) $M\times\mathbb{R} \in |aC|$ (*or* $M\times\mathbb{R}$ $|atS|$) *iff* M *is almost Kaehlerian.*

(*iv*) $M\times\mathbb{R} \in |qKC|$ (*or* $M\times\mathbb{R}$ $|qtS|$) *iff* M *is quasi-Kaehlerian.*

(*v*) $M\times\mathbb{R} \in |sC|$ *iff* M *is semi-Kaehlerian.*

(*vi*) $M\times\mathbb{R} \in |N|$ *iff* M *is Hermitian.*

(*vii*) $M\times\mathbb{R} \in |sCN|$ *iff* M *is Hermitian and semi-Kaehlerian.*

(*viii*) $M\times\mathbb{R} \in |G_1S|$ *iff* M *is a* G_1*-manifold.*

(*ix*) $M\times\mathbb{R} \in |G_2S|$ *iff* M *is a* G_2*-manifold.*

(*x*) $M\times\mathbb{R} \in |G_1sC|$ *iff* M *is a semi-Kaehler* G_1*-manifold.*

(*xi*) $M\times\mathbb{R} \in |G_2sC|$ *iff* M *is a semi-Kaehler* G_2*-manifold.*

Furthermore, $M\times\mathbb{R}$ *does not belong to* $|sS|$.

We omit the proof because it follows directly from the definitions of the different classes involved through direct calculations and in which no special devices have to be used.

Let < denote strict inclusion and □ the empty class.

THEOREM 1.2. *All the inclusions in Diagram I are strict, and in fact, we have:*

(*i*) □ < $|C|$ < $|nKC|$ < $|nC|$, $|C|$ < $|aC|$, $|C|$ < $|sCN|$.

(*ii*) $|nKC| \cup |aC| < |qKC|$.

(*iii*) $|nKC| \cup |sCN| < |G_1sC|$.

(*iv*) $|aC| \cup |sCN| < |G_2sC|$.

(*v*) $|nC| \cup |qKC| \cup |G_1sC| \cup |G_2sC| < |sC|$.

Proof: Firstly, the a. ct. m. structure on the unit sphere S^5 inherited from the almost Hermitian structure on S^6 when considered as a hypersurface [2] is a nearly cosymplectic structure which is not nearly-K-cosymplectic, and this proves $|nKC| < |nC|$.

In order to construct examples which show the remaining relations, let us consider the following manifolds:

a) $\mathbb{R}^{2n}$ endowed with the standard Kaehler structure.

b) $T(N)$, the total space of tangent bundle of a nonflat Riemannian manifold N, endowed with the standard almost Kaehler structure [8].

c) S^6, $S^2\times\mathbb{R}^4$, $N_1\times\mathbb{R}^4$ (N_1 being a nonplanar minimal surface in $\mathbb{R}^3$) endowed with the almost complex structure induced from the Cayley numbers [4].

d) $M_1 = S^6\times(N_1\times\mathbb{R}^4)$, $M_2 = T(N)\times(N_1\times\mathbb{R}^4)$, $M_3 = S^6\times T(N)\times(N_1\times\mathbb{R}^4)$ endowed with the product almost Hermitian structures.

Then, if M is an arbitrary manifold in (a), (b), (c) or (d), the product manifold $M\times\mathbb{R}$ admits an a. ct. m. structure defined by (1.1). Now, taking into account Proposition 1.1 and the results in [4],[5], we obtain

1) $\mathbb{R}^{2n+1} \in |C|$, $S^6\times\mathbb{R} \in |nKC|-|C|$, $T(N)\times\mathbb{R} \in |aC|-|C|$, $N_1\times\mathbb{R}^5 \in |sCN|-|C|$, which completes the proof of (i).
2) $S^2\times\mathbb{R}^5 \in |qKC|-(|nKC|\cup|aC|)$, which proves (ii).
3) $M_1\times\mathbb{R} \in |G_1sC|-(|nKC|\cup|sCN|)$, which proves (iii).
4) $M_2\times\mathbb{R} \in |G_2sC|-(|aC|\cup|sCN|)$, which proves (iv).
5) $M_3\times\mathbb{R} \in |sC|-(|nC|\cup|qKC|\cup|G_1sC|\cup|G_2sC|)$, which proves (v).

2. SEMI-SASAKIAN STRUCTURES

The inclusion relations between the different classes of a. ct. m. structures, which we assemble under the common denomination of semi-Sasakian structures, are graphically shown in the following diagram

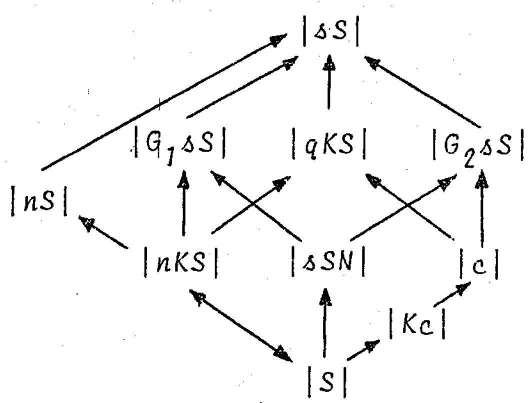

DIAGRAM II

In order to study the inclusions in this diagram, let us begin by recalling that the unit sphere S^{2r+1} inherits a Sasakian structure from the standard Kaehler structure on $\mathbb{R}^{2r+2}$ [9]. On the other hand, in [7] we have shown that a Sasakian structure is nearly-K-Sasakian and conversely. Moreover, Blair [3] gives a nearly Sasakian structure on S^5 which is not Sasakian. Therefore,

$$\square < |S| = |nKS| < |nS|.$$

Also, it is well known that a principal circle bundle over an almost Kaehler manifold has a K-contact structure, which is no Sasakian if the base is no Kaehlerian [6].

Moreover, the tangent sphere bundles are contact manifolds whose structure is not in general a K-contact structure [10]. Thus,

$$|S| < |Kc| < |c|.$$

Next, let (M,J,h) be an almost Hermitian manifold of dimension $2n$ and let M' be a $(2m'+1)$-dimensional manifold endowed with an a. ct. m. structure (φ',ξ',η',g'); on the product manifold $M\times M'$, dim $M\times M' = 2m+1$, $m=n+m'$, we define an a. ct. m. structure $(\tilde\varphi,\tilde\xi,\tilde\eta,\tilde g)$ by setting

$$\tilde\varphi(X,X') = (JX,\varphi'X'),\ \tilde\xi = \frac{m}{m'}(0,\xi'),\ \tilde\eta(X,X') = \frac{m'}{m}\eta'(X'),$$

(2.1)

$$\tilde g((X,X'),(Y,Y')) = h(X,Y)+\frac{m'^2}{m^2}g'(X',Y'),$$

where (X,X'), (Y,Y') denote arbitrary vector fields on $M\times M'$, $X,Y \in \mathfrak{X}(M)$, $X',Y' \in \mathfrak{X}(M')$. The fundamental 2-form $\tilde\Phi$ of the $(\tilde\varphi,\tilde\xi,\tilde\eta,\tilde g)$-structure satisfies

$$\tilde\Phi((X,X'),(Y,Y')) = F(X,Y) + \frac{m'^2}{m^2}\Phi'(X',Y'),$$

F being the Kaehler form of (J,h) and Φ' the fundamental 2-form of the (φ',ξ',η',g')-structure. Let $D,\nabla',\tilde\nabla$ denote the Riemannian connections of the metrics h, g', $\tilde g$, respectively; then, for $X,Y,Z \in \mathfrak{X}(M)$, $X',Y',Z' \in \mathfrak{X}(M')$, we have

$$(\tilde{\nabla}_{(X,X')}\tilde{\varphi})(Y,Y') = ((D_X J)Y,(\nabla'_{X'}\varphi')Y'),$$

$$(\tilde{\nabla}_{(X,X')}\tilde{\Phi})((Y,Y'),(Z,Z')) = (D_X F)(Y,Z)+\frac{m'^2}{m^2}(\nabla'_{X'}\Phi')(Y',Z'),$$

$$d\tilde{\Phi}((X,X'),(Y,Y'),(Z,Z')) = dF(X,Y,Z) + \frac{m'^2}{m^2}d\Phi'(X',Y',Z'),$$

$$\tilde{\delta}\tilde{\Phi}(X,X') = \delta F(X) + \delta'\Phi'(X'),$$

$$\tilde{\nabla}_{(X,X')}\tilde{\xi} = \frac{m}{m'}(0,\nabla'_{X'}\xi'),$$

$$(\tilde{\nabla}_{(X,X')}\tilde{\eta})(Y,Y') = \frac{m'}{m}(\nabla'_{X'}\eta')Y',$$

$$d\tilde{\eta}((X,X'),(Y,Y')) = \frac{m'}{m}d\eta'(X',Y'),$$

$$\tilde{\delta}\tilde{\eta} = \frac{m}{m'}\delta'\eta',$$

where δ, δ', $\tilde{\delta}$ denote the coderivatives of (M,h), (M',g'), $(M\times M',\tilde{g})$, respectively. From these identities, a straightforward calculation leads to

PROPOSITION 2.1. *Let $\tilde{M}$ denote the product manifold $M\times M'$ endowed with the a. ct. m. structure $(\tilde{\varphi},\tilde{\xi},\tilde{\eta},\tilde{g})$. Then*

(i) $\tilde{M} \in |sS|$ iff M is semi-Kaehlerian and $M' \in |sS|$.

(ii) $\tilde{M} \in |N|$ iff M is Hermitian and $M' \in |N|$.

(iii) $\tilde{M} \in |G_1S|$ iff M is a G_1-manifold and $M' \in |G_1S|$.

(iv) $\tilde{M} \in |G_2S|$ iff M is a G_2-manifold and $M' \in |G_2S|$.

Furthermore, $\tilde{M}$ does not belong to $|qKS|$.

If we now take as almost Hermitian manifold M one of the manifolds $N_1\times\mathbb{R}^4$, M_1, M_2 or M_3, which have been considered in the proof of Theorem 1.2, and as a. ct. m. manifold M' the Sasakian manifold $S^{2m'+1}$, Proposition 2.1 above allows us to check that:

$N_1\times\mathbb{R}^4\times S^{2m'+1} \in |sSN|-|S|$,

$M_1\times S^{2m'+1} \in |G_1sS|-|sSN|$,

$M_2\times S^{2m'+1} \in |G_2sS|-(|c|\cup|sSN|)$,

$M_3\times S^{2m'+1} \in |sS|-(|nS|\cup|qKS|\cup|G_1sS|\cup|G_2sS|)$.

Summing up,

THEOREM 2.2. *For the classes of semi-Sasakian structures in Diagram II we have:*

(i) $\square < |S| = |nKS| < |nS|$, $|S| < |Kc| < |c|$, $|S| < |sSN|$.

(ii) $|sSN| < |G_1sS|$.

(iii) $|c|\cup|sSN| < |G_2sS|$.

(iv) $|nS|\cup|qKS|\cup|G_1sS|\cup|G_2sS| < |sS|$.

NOTE. Remark that this theorem does not prove the strictness of the inclusion $|c|\subset|qKS|$, which remains as an open question.

3. THE REMAINING CLASSES

Let us consider the following diagram

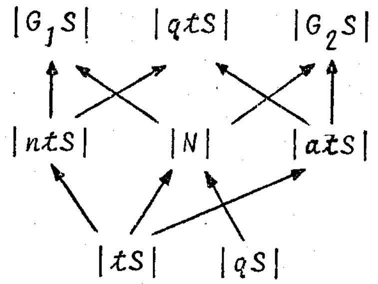

DIAGRAM III

In this section, we shall be concerned with proving not only the strictness of the inclusions in this diagram, but also the strictness of all the possible inclusions of semi-cosymplectic and semi-Sasakian classes in the classes which are represented in it.

First of all, we can state

THEOREM 3.1. *(i)* $|sCN| \cup |sSN| \cup |tS| \cup |qS| < |N|$.
(ii) $|G_1sC| \cup |G_1sS| \cup |ntS| \cup |N| < |G_1S|$.
(iii) $|G_2sC| \cup |G_2sS| \cup |atS| \cup |N| < |G_2S|$.

Proof: (i). Let us consider the Calabi-Eckmann manifolds $S^{2k+1}\times S^{2\ell+1}$ for $k,\ell > 1$; then, from Propositions 1.1 and 2.1, $S^{2k+1}\times S^{2\ell+1}\times\mathbb{R}$ and $S^{2k+1}\times S^{2\ell+1}\times S^{2r+1}$ are normal a. ct. m. manifolds which do not belong to $|sCN| \cup |sSN| \cup |tS| \cup |qS|$.

(ii) and (iii). Let us denote $\overline{M}_1$ and $\overline{M}_2$ the manifolds which are obtained from the manifolds M_1 and M_2 in the proof of Theorem 1.2 by making a (nontrivial) conformal change of the metric. Then, taking into account Theorem 5.2 in [5], which classifies the almost Hermitian structure of $\overline{M}_1$ and $\overline{M}_2$, and by using the results in Proposition 1.1, it easily follows:

$$\overline{M}_1\times\mathbb{R} \in |G_1S| - (|G_1sC| \cup |G_1sS| \cup |ntS| \cup |N|),$$

$$\overline{M}_2\times\mathbb{R} \in |G_2S| - (|G_2sC| \cup |G_2sS| \cup |atS| \cup |N|).$$

Now, let M be a $2n$-dimensional manifold with almost Hermitian structure (J,h); on the product manifold $M\times\mathbb{R}$ we consider again the (φ,ξ,η,g)-structure defined by (1.1). A conformal change of this a. ct. m. structure on $M\times\mathbb{R}$ can be defined by setting

$$(3.1)\quad \varphi^\circ = \varphi, \quad \xi^\circ = e^{-\tau}\xi, \quad \eta^\circ = e^{\tau}\eta, \quad g^\circ = e^{2\tau}g,$$

where τ denotes the canonical projection $M\times\mathbb{R}\longrightarrow\mathbb{R}$. Then, $d\tau = \eta$ and the fundamental 2-form Φ° of the $(\varphi^\circ,\xi^\circ,\eta^\circ,g^\circ)$-

structure on $M\times\mathbb{R}$ is given by

$$\Phi^\circ = e^{2\tau}\Phi.$$

Let ∇, δ $(\nabla^\circ,\delta^\circ)$ denote the Riemannian connection and the coderivative of g (g°), $\overline{X} = (X, a\frac{d}{dt})$, $\overline{Y} = (Y, b\frac{d}{dt})$, $\overline{Z} = (Z, c\frac{d}{dt})$ arbitrary vector fields on $M\times\mathbb{R}$; then,

$$(\nabla^\circ_{\overline{X}}\varphi^\circ)\overline{Y} = (\nabla_{\overline{X}}\varphi)\overline{Y} - \eta(\overline{Y})\varphi(\overline{X}) - g(\overline{X},\varphi\overline{Y})\xi,$$

$$(\nabla^\circ_{\overline{X}}\Phi^\circ)(\overline{Y},\overline{Z}) = e^{2\tau}\{(\nabla_{\overline{X}}\Phi)(\overline{Y},\overline{Z}) - \eta(\overline{Y})\Phi(\overline{X},\overline{Z}) + \Phi(\overline{X},\overline{Y})\eta(\overline{Z})\},$$

$$d\Phi^\circ = e^{2\tau}(2\eta\wedge\Phi + d\Phi),$$

$$\delta^\circ\Phi^\circ = \delta\Phi,$$

$$\nabla^\circ_{\overline{X}}\xi^\circ = e^{-\tau}(\overline{X} - \eta(\overline{X})\xi),$$

$$(\nabla^\circ_{\overline{X}}\eta^\circ)\overline{Y} = e^{\tau}g(\varphi\overline{X},\varphi\overline{Y}),$$

$$d\eta^\circ = 0,$$

$$\delta^\circ\eta^\circ = -2ne^{-\tau}.$$

In the sequel, $M\times\mathbb{R}$ denotes this product manifold with the a. ct. m. structure (φ,ξ,η,g) and $(M\times R)^\circ$ denotes the same manifold with the a. ct. m. structure $(\varphi^\circ,\xi^\circ,\eta^\circ,g^\circ)$.

From the identities above, a straightforward calculation leads to

PROPOSITION 3.2. *Suppose* $M\times\mathbb{R} \in |sC|$. *Then* $(M\times\mathbb{R})^\circ$ *does not belong to* $|sC| \cup |sS|$, *and moreover*

(*i*) $(M\times\mathbb{R})^\circ \in |tS|$ *iff* $M\times\mathbb{R} \in |C|$.

(*ii*) $(M\times\mathbb{R})^\circ \in |ntS|$ *iff* $M\times\mathbb{R} \in |nKC|$.

(*iii*) $(M\times\mathbb{R})^\circ \in |atS|$ *iff* $M\times\mathbb{R} \in |aC|$.

(*iv*) $(M\times\mathbb{R})^\circ \in |qtS|$ *iff* $M\times\mathbb{R} \in |qKC|$.

The strictness of the inclusions between the remaining classes of a. ct. m. manifolds is now an easy consequence of Proposition 3.2. In fact, we can state

THEOREM 3.3. (*i*) $|C| \cup |S| < |tS|$.

(*ii*) $|nKC| \cup |tS| < |ntS|$.

(*iii*) $|aC| \cup |c| \cup |tS| < |atS|$.

(*iv*) $|qKC| \cup |qKS| \cup |ntS| \cup |atS| < |qtS|$.

Proof: Let us consider the manifolds $\mathbb{R}^{2n+1}$, $S^6\times\mathbb{R}$, $T(N)\times\mathbb{R}$ and $S^2\times\mathbb{R}^5$ endowed with their a. ct. m. structures as defined in §1. Then

$(\mathbb{R}^{2n+1})^\circ \in |tS| - (|C| \cup |S|)$,

$(S^6\times\mathbb{R})^\circ \in |ntS| - (|nKC| \cup |tS|)$,

$(T(N)\times\mathbb{R})^\circ \in |atS| - (|aC| \cup |c| \cup |tS|)$,

$(S^2\times\mathbb{R}^5)^\circ \in |qtS| - (|qKC| \cup |qKS| \cup |ntS| \cup |atS|)$.

REMARK 1. In [7] we have introduced the almost-K-contact structures ($|aKc|$) as those (φ,ξ,η,g)-structures which verify $\nabla_\xi\varphi = 0$ and proved the inclusion $|N| \cup |qKC| \cup |qKS| \subset |aKc|$; actually this inclusion is also strict. To prove this, note that all the product manifolds $M\times\mathbb{R}$, $M\times M'$ and $(M\times\mathbb{R})^\circ$, as considered in §1, §2 and §3, respectively, belong to the class $|aKc|$, but in fact, $M_3\times\mathbb{R}$, $M_3\times S^{2r+1}$, $(S^2\times\mathbb{R}^5)^\circ$ do not belong to $|N| \cup |qKC| \cup |qKS|$.

REMARK 2. In [7] we have studied some relations between the classes of trans-Sasakian and quasi-Sasakian structures. As it is known, both classes contain those of cosymplectic and Sasakian structures, that is $|C| \cup |S| \subset |tS| \cap |qS|$. However, $|tS|$ and $|qS|$ are not related by inclusion. For instance, if (M,J,h) is a Kaehler manifold and (φ',ξ',η',g') is a Sasakian structure on M', then the a. ct. m. structure (φ,ξ,η,g) defined on $M\times M'$ by (2.1) is quasi-Sasakian but not trans-Sasakian. Conversely, let (φ,ξ,η,g) be the a. ct. m. structure on $\mathbb{R}^{2n+1} \equiv \mathbb{R}^{2n}\times\mathbb{R}$ defined by (1.1) from the standard Kaehler structure on $\mathbb{R}^{2n}$; if we make the conformal change given by (3.1), the a. ct. m. structure $(\varphi^\circ,\xi^\circ,\eta^\circ,g^\circ)$ on $\mathbb{R}^{2n+1}$ is trans-Sasakian but not quasi-Sasakian.

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