Real Hypersurfaces in Hermitian Symmetric Spaces and Related Topics

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Symmetry and Shape
Celebrating the 60th birthday of Prof. J. Berndt
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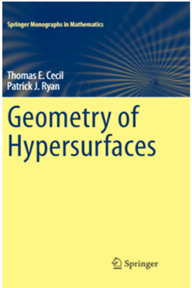
“Springer Monographs in Mathematics - **Geometry of Hypersurfaces**”

- By T.E. Cecil & P.J. Ryan, Springer, ISBN: 978-1-4939-3245-0



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All the material we have discussed in this book falls under the heading of
“Hypersu … ed in
complexit … space
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have been … perty,
and speci … r, the
curvature … r and
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lasses of hypersurfaces is that they are tubes over their local submanifolds, or at
ast share many of the algebraic properties of such tubes.

The most active area of study over the past decade has been that of hypersurfaces in the next most complicated ambient spaces, the complex two-plane Grassmannians (see Section 7.4). This topic was introduced by Berndt and Suh [29, 37] and approximately 100 papers have appeared in which many of the problems discussed in Chapters 8 and 9 have been studied in this new context. Unfortunately, limitations of time and space do not permit us to discuss these results in the current volume.

J. Berndt(King's College London) & Y.J. Suh(Kyungpook Nat'l Univ.) **에 의해 공동연구된 복소 두 평면 그라스만 다양체의 실초곡면에 관한 연구를 차기연구(further research)분야로 언급**

※ Geometry of Hypersurfaces (by Cecil and Ryan)에서 발췌 ※

Problem 1

Classify all of homogeneous hypersurfaces in HSS.

Problem 2

If M is a connected hypersurface with isometric Reeb flow in HSS $\bar{M}$, then M becomes homogeneous ?
Answer: Yes, For $G_2(\mathbf{C}^{m+2})$ Berndt and Suh: Monat(2002), For Q^m Berndt and Suh: IJM(2013), and Q^{m*} Suh: CCM(2018), For HSS, Berndt and Suh: CCM(2020).

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Definition

A hypersurface M: Isometric Reeb Flow $\iff$ $\mathcal{L}_\xi g = 0$ $\iff$ $g(d\phi_t X, d\phi_t Y) = g(X, Y)$ for any $X, Y \in \Gamma(M)$, where ϕ_t denotes a one parameter group, which is said to be an isometric Reeb flow of M, defined by

$$\frac{d\phi_t}{dt} = \xi(\phi_t(p)), \quad \phi_0(p) = p, \dot{\phi}_0(p) = \xi(p).$$

Note)

$\mathcal{L}_\xi g = 0 \iff \nabla_j \xi_i + \nabla_i \xi_j = 0$, $\nabla \xi$: skew-symmetric $\iff$ $g(\nabla_X \xi, Y) + g(\nabla_Y \xi, X) = 0 \iff g((\phi S - S\phi)X, Y) = 0$ for any $X, Y \in \Gamma(M)$.

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Herm. Symm. Spaces with Isometric Reeb Flow

- (A) The complex Grassmann manifolds $G_k(\mathbf{C}^{r+1}) = SU_{r+1}/S(U_k U_{r+1-k})$,
- (B) The complex quadrics $Q^{r+2} = SO_{r+2}/SO_r SO_2$, $(r \geq 3)$,
- (C) The complex Lag. Grassmann Sp_r/U_r, $r \geq 3$, the set of all complex r-dim $\mathbf{C}^r$ in $\mathbf{H}^r$,
- (D) The symmetric spaces SO_{2r}/U_r, $(r \geq 5)$, the space of all almost complex structures on $\mathbf{R}^{2r}$,
- (E_6) The complexified Cayley proj. plane $E_6/Spin_{10}U_1$,
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For $G_k(\mathbf{C}^{r+1})$ in (A) with $k = 1$ by Okumura (TAMS. 1976), $k = 2$, by Berndt and Suh (Monat. 2002), and for $Q^{r+2} = SO_{r+2}/SO_r SO_2$, also by Berndt and Suh (IJM. 2013).

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Complex k Grassmannians $G_k(\mathbf{C}^{r+1})$

$$\bar{M} = G/K = SU(r+1)/SU(k)SU(r+1-k) = G_k(\mathbf{C}^{r+1}),$$

$$\triangle^+ = \{\epsilon_i - \epsilon_j | i < j, 1 \leq i, j \leq r+1\},$$

$$\Lambda = \{\alpha_1, \cdots, \alpha_r\}, \quad \alpha_i = \epsilon_i - \epsilon_{i+1},$$

$$\triangle^+ = \{\alpha_\nu + \cdots + \alpha_\mu | 1 \leq \nu < \mu \leq r\},$$

$$\triangle^+_{\bar{M}} = \{\alpha_{\nu\mu} \in \triangle^+ | 1 \leq \nu \leq k \leq \mu \leq r\},$$

where $\alpha_{\nu\mu} = \alpha_\nu + \cdots + \alpha_k + \cdots + \alpha_\mu$.

Note that $|\triangle_M^+| = k(r+1-k) = \frac{1}{2}\mathrm{dim}(\mathrm{G_k}(\mathbf{C}^{\mathrm{r+1}}))$ and

$$T_o\bar{M} = \bigoplus_{\alpha\in\triangle_{\bar{M}}^+} \mathbf{C}u_\alpha.$$

Now we define two subsets

$$\triangle_M^+(0) = \{\alpha_{\nu,\mu}\in\triangle_M^+ | \nu > 1 \text{ and } \mu < \mathrm{r}\},$$

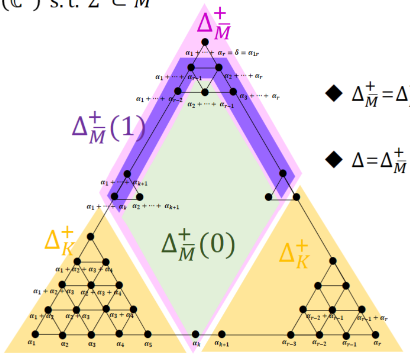
$$\triangle_M^+(1) = \{\alpha_{\nu,\mu}\in\triangle_M^+ | \nu = 1 \text{ or } \mu = \mathrm{r}\} - \{\alpha_{1,\mathrm{r}}\}.$$

Then it follows that

$$\triangle_M^+ = \triangle_M^+(0)\cup\triangle_M^+(1)\cup\{\delta\},$$

where $\delta = \alpha_{1,r} = \alpha_1 + \cdots + \alpha_r \in \triangle_M^+$ denotes the highest root.

- $\bar{M} = G_k(\mathbb{C}^{r+1})$
- $\Sigma = G_k(\mathbb{C}^r)$ s.t. $\Sigma \subset \bar{M}$



- $\Delta^+_{\bar{M}} = \Delta^+_{\bar{M}}(0) \cup \Delta^+_{\bar{M}}(1) \cup \{\delta\}$
  where $\delta = \alpha_1 + \alpha_2 + \cdots + \alpha_r$
- $\Delta = \Delta^+_{\bar{M}} \cup \Delta^+_K$

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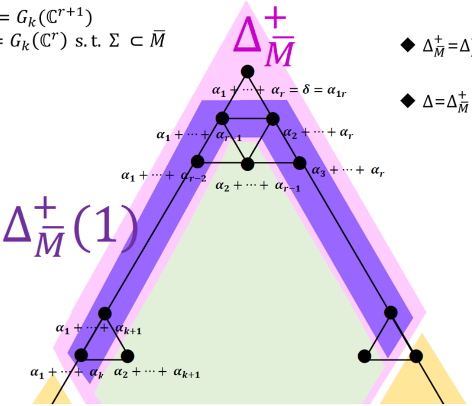

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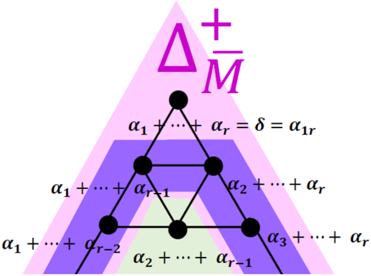

Now let us denote by M_t the tubes of radius t around Σ, with t sufficiently small. We choose $\frac{u_\delta}{|u_\delta|}$ for the direction of the normal geodesic in $\bar{M}$ with $\gamma(0) = o$ and $\dot{\gamma}(0) = \frac{u_\delta}{|u_\delta|}$.

We consider the End($\gamma^\perp$)-valued Jacobi differential equation

$$Y'' + \bar{R}_\gamma^\perp \circ Y = 0.$$

Then the shape operator $S(t)$ of M_t with respect to $-\dot{\gamma}(t)$ is given by

$$S(t) = D'(t) \circ D^{-1}(t).$$

By the expression of the shape operator, we can assert that the Reeb flow of a tube over a complex totally geodesic Grassmannian $G_k(\mathbf{C}^r)$ in $G_k(\mathbf{C}^{r+1})$ is isometric, that is $S\phi = \phi S$.

Proposition 1.4.

Let M_t be the tube of radius $0 < t < \frac{\pi}{\sqrt{2}}$ around the totally geodesic $\Sigma = G_k(\mathbf{C}^r)$ in $\bar{M} = G_k(\mathbf{C}^{r+1})$. Then

- 1. M_t is a Hopf hypersurface.
- 2. Principal curvature spaces and multiplicities are given

principal curvature	multiplicity	eigenspace
$\alpha = \sqrt{2}\cot(\sqrt{2}t)$	1	$T_\alpha = J\frac{u_\delta}{\lvert u_\delta \rvert}$
$\beta = \frac{1}{\sqrt{2}}\cot(\frac{1}{\sqrt{2}}t)$	$2(k-1)$	$T_\beta = \nu_o\Sigma$
$\lambda = -\frac{1}{\sqrt{2}}\tan(\frac{1}{\sqrt{2}}t)$	$2(r-k)$	$T_\lambda = T_o^0\Sigma$
$\mu = 0$	$2(k-1)(r-k)$	$T_\mu = T_o^1\Sigma$

- 3. The Reeb flow on M_t is isometric.

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For $k = 1$ (Okumura, Trans AMS, 1976), and $k = 2$ (Berndt and Suh, Monat. für Math. 2002). These geometric structures help tremendously for explicit tensor calculus.

This time, by using structure theory of real and complex semi-simple Lie algebras we prove the following

Theorem 1.5. (Berndt and Suh, CCM, 2020)

Let M be a connected orientable real hypersurface in complex Grassmannians $G_k(\mathbf{C}^{r+1})$. Then the Reeb flow on M is isometric $\iff$ M is a tube over a complex totally geodesic Grassmannian $G_k(\mathbf{C}^r)$ in $G_k(\mathbf{C}^{r+1})$.

Motivated by the above facts and all of documents mentioned above, we have the following

Theorem A (Berndt and Suh, CCM, 2020)

Let M be a real hypersurface in Hermitian symmetric space $\bar{M}$ of compact type. If the Reeb flow on M is isometric, then M is congruent to an open part of a tube of radius $0 < t < \frac{\pi}{\sqrt{2}}$ around the totally geodesic submanifold Σ in $\bar{M}$, where

- 1. $\bar{M} = \mathbf{C}P^{r+1}$ and $\Sigma = \mathbf{C}P^k$, $r \geq 1$, $0 \leq k \leq r$,
- 2. $\bar{M} = G_k(\mathbf{C}^{r+1})$ and $\Sigma = G_k(\mathbf{C}^r)$, $k \geq 2$, $r \geq 3$,
- 3. $\bar{M} = SO_{2k+2}/SO_{2k}SO_2$ and $\Sigma = \mathbf{C}P^k$, $k \geq 3$,
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Conversely, the Reeb flow of any such tube is isometric.

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Definition of contact hypersurfaces

Definition

A hypersurface M in m-dim. Kaehler manifold $\bar{M}$ is contact $\iff$ there exists a non-vanishing smooth function ρ on M such that $d\eta = \rho\omega$. Then it is clear that $\eta \wedge (d\eta)^{m-1} \neq 0$.

Note)

Here $d\eta = \rho\omega \iff d\eta(X, Y) = \rho\omega(X, Y) = \rho g(\phi X, Y)$.

Here the 2-form $d\eta$ is defined by

$$\begin{aligned} 2d\eta(X, Y) &= X(\eta(Y)) - Y(\eta(X)) - \eta([X, Y]) \\ &= g((S\phi + \phi S)X, Y) \end{aligned}$$

Definition of contact hypersurfaces

Definition

A hypersurface M in m-dim. Kaehler manifold $\bar{M}$ is contact $\iff$ there exists a non-vanishing smooth function ρ on M such that $d\eta = \rho\omega$. Then it is clear that $\eta \wedge (d\eta)^{m-1} \neq 0$.

Note)

Here $d\eta = \rho\omega \iff d\eta(X, Y) = \rho\omega(X, Y) = \rho g(\phi X, Y)$.

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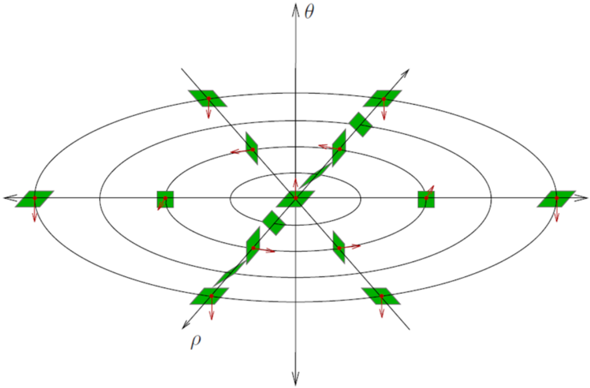

Contact structure on S^3.

A Key Proposition in Kaehler Manifold

A contact hypersurface in a Kaehler manifold is a real hypersurface satisfying the condition:

$$S\phi + \phi S = k\phi, \quad k = 2\rho \neq 0 : \quad \text{constant}$$

Proposition 2.3. (Berndt and Suh, Proc. AMS., 2015)

Let M be a contact hypersurface in a Kaehler manifold. Then the following statements are equivalent:

- (i) The function α is constant,
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Complex Hyperbolic Space CH^m

Theorem 2.1. (Vernon, Tohoku, Math. J., 1987)

Let M be a connected contact real hypersurface in $\mathbf{C}H^m$. Then

- (A) M is a tube around $\mathbf{C}H^{m-1}$ in $\mathbf{C}H^m$,
- (B) M is a tube around a totally real $\mathbf{R}H^m$ in $\mathbf{C}H^m$,
- (C) geodesic hypersphere
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Note. Every complete contact hypersurface in a complex space form becomes a homogeneous hypersurface.

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Complex Two-Plane Grassmannian $G_2(\mathbf{C}^{m+2})$

Theorem 2.2. (Suh, Monat. fur Math., 2006)

Let M be a contact real hypersurface in $G_2(\mathbf{C}^{m+2})$, $m \geq 3$, with constant mean curvature. Then

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Non-Compact Grassmannian $SU_{2,m}/S(U_2U_m)$

A real hypersurface M in $SU_{2,m}/S(U_2U_m)$ is said to be a *contact* if and only if there exists a non-zero constant function ρ defined on M such that

$$\phi S + S\phi = k\phi, \quad k = 2\rho.$$

This formula means that for any vector fields X, Y on M

$$g((\phi S + S\phi)X, Y) = 2d\eta(X, Y),$$

where $d\eta$ of the 1-form η is defined by

$$2d\eta(X, Y) = (\nabla_X \eta)Y - (\nabla_Y \eta)X.$$

Then we give a classification of *contact* real hypersurfaces in noncompact complex two-plane Grassmannian $SU_{2,m}/S(U_2U_m)$ as follows:

Theorem 2.3. (Berndt, Lee and Suh, Int. J. Math., 2013)

Let M be a connected contact hypersurface in $SU_{2,m}/S(U_2U_m)$, $m \geq 3$. If the Reeb function α corresponding to the Reeb vector field ξ is constant along the curve of ξ, then M is locally congruent to one of the following:

- (i) a horosphere with singular center at infinity of type $JX \perp \mathfrak{J}X$,
- (ii) (only if $m = 2k$ is even) a tube around the totally geodesic embedding of the quaternionic hyperbolic space $\mathbb{H}H^k$.

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A Key Proposition in Complex Quadric Q^m

For $M \subset Q^m$ we know that

$$\bar{R}_N JN = \bar{R}(JN, N)N = 4JN + 2\cos(2t)AJN.$$

Then JN: eig. vector of $\bar{R}_N \quad \Leftrightarrow \quad t = \frac{\pi}{4}$ or N: $\mathfrak{A}$-principal.

Proposition 2.4

Let M be a contact hypersurface in Q^m (resp. Q^{m*}), $m \geq 3$. Then the following statements are equivalent:

- (i) JN is an eigenvector of $\bar{R}_N = \bar{R}(\cdot, N)N$,
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Contact Hypersurfaces of Type (B)

By virtue of key Propositions and some remarks mentioned above, first we give a classification of contact hypersurfaces in Q^m as follows:

Theorem 3.1. (Berndt and Suh, Proc. AMS., 2015)

Let M be a connected real hypersurface with constant mean curvature in complex quadric Q^m, $m \geq 3$. Then M is contact $\iff$ M is an open part of a tube of radius $0 < r < \frac{\pi}{2\sqrt{2}}$ around the sphere S^m embedded in Q^m.

Complex Hyperbolic Quadric Q^{m*}

We realize the complex hyperbolic quadric $Q^{m*} \simeq SO_{2,m}/SO_2SO_m$. As $Q^{1^*} \simeq \mathbb{R}H^2 = SO_{1,2}/SO_2$, and $Q^{2^*} \simeq \mathbb{C}H^1 \times \mathbb{C}H^1$, we suppose $m \geq 3$. Let $G := SO_{2,m}$ be a transvection group of Q^{m*} and $K := SO_2SO_m$ be the isotropy group of Q^{m*} at $p_0 := eK \in Q^{m*}$. Then

$$\sigma : G \to G,\ g \mapsto sgs^{-1} \quad \text{with} \quad s := \begin{pmatrix} -1 & & & & & \\ & -1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{pmatrix}$$

is an involution of G with $\mathrm{Fix}(\sigma)_0 = K$, and therefore $Q^{m*} = G/K$.

Theorem 3.2.

The tube M around the totally geodesic ${Q^{m-1}}^*$ in ${Q^m}^*$ exists for every radius $r > 0$. For M the following statements hold:

(1) Every normal vector N of M is $\mathfrak{A}$-principal.

(2) M has constant principal curvatures. Then the principal curvatures and the principal curvature spaces are

principal curvature	curvature space	multi
$\lambda = 0$	$J(V(A) \ominus \mathbb{R}N)$	$m-1$
$\mu = -\sqrt{2}\tanh(\sqrt{2}r)$	$V(A) \ominus \mathbb{R}N$	$m-1$
$\alpha = -\sqrt{2}\coth(\sqrt{2}r)$	$\mathbb{R}JN$	1

(3) M is contact, that is, $S\phi + \phi S = k\phi$.

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The *horosphere* with *center at infinity* $\gamma(\infty)$ through some point $p \in \bar{M}$ is defined as

$$C(p,\gamma(\infty)) = \left\{ q \in \bar{M} \;\middle|\; \lim_{t\to\infty}(d(q,\gamma(t)) - d(p,\gamma(t))) = 0 \right\}.$$

We consider $\bar{M} = G/K$ with the "origin" $o := eK \in \bar{M}$, the Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ and a Cartan subalgebra $\mathfrak{a} \subset \mathfrak{p}$. Further consider the root system $\Sigma \subset \mathfrak{a}^*$ and for a positive root system $\Sigma^+ \subset \Sigma$, $\mathfrak{n} := \bigoplus_{\lambda\in\Sigma^+} \mathfrak{g}_\lambda$ is a nilpotent subalgebra of $\mathfrak{g}$, and

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{a} \oplus \mathfrak{n}$$

is an Iwasawa decomposition of $\mathfrak{g}$.

Now suppose that a unit vector $H \in \mathfrak{a}$ is given. Then

$$\mathfrak{s}_H := (\mathfrak{a} \ominus \mathbb{R}H) \oplus \mathfrak{n},$$

is a solvable Lie subalgebra of $\mathfrak{g}$.

Let S_H be the connected subgroup of AN with Lie algebra $\mathfrak{s}_H$. Then the orbits of the action of S_H on $\bar{M}$ are the horospheres of $\bar{M}$ with the center at infinity $\gamma_H(\infty)$, where γ_H is the geodesic with $\gamma_H(0) = o$ and $\dot{\gamma}_H(0) = H$ (and where we identify $\mathfrak{p}$ with $T_e\bar{M}$ in the usual manner). In particular we have

$$C(o, \gamma_H(\infty)) = S_H \cdot o,$$

where the shape operator of $C(o, \gamma_H(\infty))$ with respect to the unit normal vector H is given by $\mathrm{ad}(H)|\mathfrak{s}_H$

Geometric Structures of Horosphere

Theorem 3.4

Let M be a horosphere in Q^{m*} with its center at infinity being given by an $\mathfrak{A}$-principal geodesic γ. Then the following statements hold:

(1) Every normal vector N of M is $\mathfrak{A}$-principal.

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principal curvature	curvature space	multi
0	$J(V(A) \ominus \mathbb{R}N)$	$m-1$
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Contact hypersurfaces of type (B) in Q^{m*}

Theorem 3.5. Suh and Klein, Anali di Mate, 2019

Let M be a connected real hypersurface with cmc in Q^{m*}, $m \geq 3$. Then M is contact if and only if M is an open part of one of the following

- (i)the tube of radius $r \in \mathbf{R}_+$ around $Q^{(m-1)^*}$ in Q^{m*},
- (ii)the tube of radius $r \in \mathbf{R}_+$ around $\mathbf{R}H^m$ in Q^{m*} as a real form of Q^{m*}.
- (iii) a horosphere in Q^{m*} with $\mathfrak{A}$-principal in Q^{m*}.

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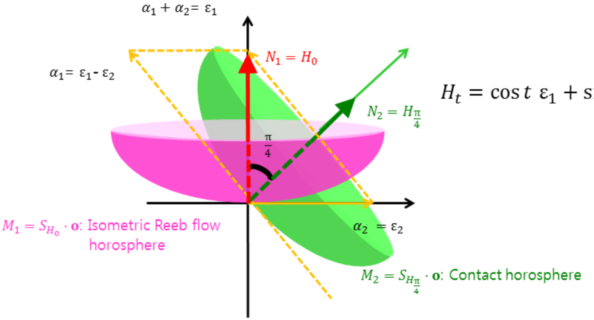
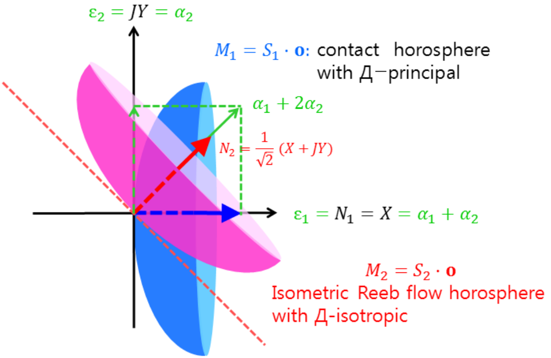
Horosphers in Complex Hyperbolic Grassmannians



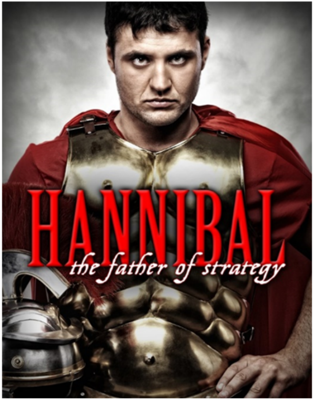
$$H_t = \cos t\ \varepsilon_1 + \sin t\ \varepsilon_2 \in Д$$

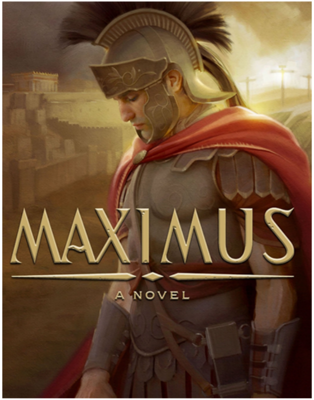
Horosphers in Complex Hyperbolic Quadrics



TWO METHODS

(Tensor Analysis) (Lie Algebraic Method)





Contact Hypersurfaces in HSSM

Proposition 4.1.

Let M be a contact hypersurface of an irreducible Hermitian symmetric space $\bar{M}$. Then we have $d\alpha(JN) = 0$.

Proposition 4.2.

Let M be a contact hypersurface of an irreducible Hermitian symmetric space $\bar{M}$, $n > 2$. If $SX = \lambda X$ with $X \in \mathcal{C}$, then

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We have the following

Proposition 4.3.

Let M be a connected orientable real hypersurface with geodesic Reeb flow in an Hermitian symmetric space $\bar{M}$. Then

$$\begin{aligned} d\alpha(JN)g((S\phi+\phi S)X,Y) &= \eta(X)g(\bar{R}_N JN, SY) \\ &\quad -\eta(Y)g(\bar{R}_N JN, SX) - \alpha\eta(X)g(\bar{R}_N JN, Y) \\ &\quad +\alpha\eta(Y)g(\bar{R}_N JN, X) - 3g(\bar{R}_N JY, JSX) \\ &\quad +3g(\bar{R}_N JX, JSY) - g(\bar{R}_N Y, SX) + g(\bar{R}_N X, SY). \end{aligned}$$

For the special case of contact hypersurfaces Proposition 4.3 implies

Proposition 4.4.

Let M be a connected orientable contact hypersurface in an Hermitian symmetric space $\bar{M}$. Then

$$\begin{aligned}\rho d\alpha(JN) =& g(\bar{R}(X,N)N, JSX) + g(\bar{R}(JX,N)N, SX)\\ &- 2\rho g(\bar{R}(JX,N)N, X).\end{aligned}$$

for all $X \in \mathcal{C}$ with $\|X\| = 1$.

Summing up all of Propositions 4.1, 4.2, 4.3, and 4.4, we can assert the folowing

Theorem 4.1. Berndt and Suh, 2020

Let M be a connected orientable contact hypersurface in an irreducible Hermitian symmetric space $\bar{M}^n$, $n > 2$. Then the following statements hold:

(i) α is constant;

(ii) M has constant mean curvature;

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Contact Conjecture 4.2

Theorem 4.2. (Berndt and Suh)

Let M be a connected real hypersurface of an irreducible Hermitian symmetric space $\bar{M}$ of compact type and $\dim_{\mathbb{C}}(\bar{M}) \geq 3$. Then M is a contact hypersurface of $\bar{M}$ if and only if M is a geodesic hypersphere in $\mathbb{C}P^m$ or an open part of a tube of radius $0 < r < \frac{\pi}{\sqrt{8}}$ around the real form Σ of $\bar{M}$, where ;

- (i) $\Sigma = \mathbb{R}P^k$ and $\bar{M} = \mathbb{C}P^k$ $(k \geq 3)$
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Theorem 4.2.1. (Berndt and Suh)

Let M be a connected real hypersurface of an irreducible Hermitian symmetric space $\bar{M}$ of compact type and $m = \dim_{\mathbb{C}}(\bar{M}) \geq 3$. Then M is a contact hypersurface of $\bar{M}$ if and only if M is an open part of a tube around a Kähler C-space Σ which can be embedded as a complex hypersurface into $\bar{M}$:

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Kobayashi and Nagano's Work

S.Kobayashi and T. Nagano (J. of Math. and Mechanics, Vol.13-5(1964)) have asserted some totally geodesic and totally real embedding Σ in $\bar{M}$ as follows:

- $\mathfrak{g} = \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1$ such that

$$[\mathfrak{g}_{-1}, \mathfrak{g}_{-1}] = 0, [\mathfrak{g}_{-1}, \mathfrak{g}_0] \subset \mathfrak{g}_{-1}, [\mathfrak{g}_{-1}, \mathfrak{g}_1] \subset \mathfrak{g}_0$$

$$[\mathfrak{g}_0, \mathfrak{g}_0] \subset \mathfrak{g}_0, [\mathfrak{g}_0, \mathfrak{g}_1] \subset \mathfrak{g}_1, [\mathfrak{g}_1, \mathfrak{g}_1] = 0$$

- There exists an element $Z \in \mathfrak{c}(\mathfrak{g}_0)$ such that

$$[Z, X] = ad(Z)X = -X \text{ for any } X \in \mathfrak{g}_{-1},$$

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- There exists E. Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ such that

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- Let $\mathfrak{g}_u = \mathfrak{k} \oplus i\mathfrak{p}$ be a compact real form. Then it follows that

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$$\mathfrak{g}_{-1} \oplus \mathfrak{g}_1 = (\mathfrak{g}_{-1} \oplus \mathfrak{g}_1) \cap \mathfrak{k} \oplus (\mathfrak{g}_{-1} \oplus \mathfrak{g}_1) \cap \mathfrak{p}.$$

- $$Z \in \mathfrak{g}_0 \cap \mathfrak{p}.$$

- Let $\mathfrak{g}_u = \mathfrak{k} \oplus i\mathfrak{p}$ be a compact real form. Then it follows that

$$\mathfrak{g}_u = \mathfrak{k}_u \oplus \mathfrak{m}_u,$$

$$[\mathfrak{k}_u, \mathfrak{k}_u] \subset \mathfrak{k}_u, [\mathfrak{k}_u, \mathfrak{m}_u] \subset \mathfrak{m}_u, [m_u, m_u] \subset \mathfrak{k}_u.$$

Here the subalgebra $\mathfrak{k}_u$ corresponding to the Lie group $K_u = \{k \in G_u | Ad(k)Z = Z\}$ and $\mathfrak{m}_u$ are respectively given by

- $$\mathfrak{k}_u = \mathfrak{k} \cap \mathfrak{g}_0 \oplus i(\mathfrak{p} \cap \mathfrak{g}_0)$$
- $$\mathfrak{m}_u = \mathfrak{k} \cap (\mathfrak{g}_{-1} \oplus \mathfrak{g}_1) \oplus i(\mathfrak{p} \cap (\mathfrak{g}_{-1} \oplus \mathfrak{g}_1)$$
- $$\mathfrak{k} = (\mathfrak{k} \cap \mathfrak{g}_0) \oplus (\mathfrak{k} \cap (\mathfrak{g}_{-1} \oplus \mathfrak{g}_1) = \mathfrak{k}_0 \oplus \mathfrak{m},$$
  $$[\mathfrak{k}_0, \mathfrak{k}_0] \subset \mathfrak{k}_0, [\mathfrak{k}_0, \mathfrak{m}] \subset \mathfrak{m}, [\mathfrak{m}, \mathfrak{m}] \subset \mathfrak{k}_0$$
- $$H_0 = -iZ \in \mathfrak{c}(\mathfrak{k}_u),$$

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- $$H_0 = -iZ \in \mathfrak{c}(\mathfrak{k}_u),$$

Hence $J = ad(H_0)|\mathfrak{m}_u : \mathfrak{m}_u = T_0\bar{M} \rightarrow \mathfrak{m}_u = T_0\bar{M}$ is a complex structure on $\mathfrak{m}_u = \mathfrak{m} \oplus Jm$, because for any $X \in \mathfrak{m}_u$

$$J^2X = ad(H_0)^2X = -[Z, [Z, X]] = -ad(Z)^2X = -X.$$

We can write Σ as a homogeneous space $\Sigma = G/U$. The Lie algebra $\mathfrak{u}$ of U is a parabolic subalgebra of $\mathfrak{g}$.

$$\Sigma = G/U, \quad \bar{M} = \bar{G}/\bar{U}, \quad \bar{\mathfrak{g}} = \mathfrak{g}^C = \mathfrak{g} \oplus i\mathfrak{g}$$

$$\Sigma = KAN/K_0AN = K/K_0, \quad \bar{M} = G_u/K_u, \quad \mathfrak{g}_u = \mathfrak{k} \oplus i\mathfrak{p},$$

where the isotropic subgroups are given by

$$K_0 = \{k \in K | Ad(k)Z = Z\}$$

$$K_u = \{k \in G_u | Ad(k)Z = Z\}$$

Then we can write

$$\Sigma = K/K_o \text{ and } \bar{M} = G_u/K_u.$$

Explicitly, we have

$$\bar{M} = \begin{cases} Q^k = SO_{k+2}/SO_k SO_2 \\ \mathbb{C}P^k = SU_{k+1}/S(U_k U_1) \\ \mathbb{C}P^k \times \mathbb{C}P^k = (SU_{k+1} \times SU_{k+1})/(S(U_k U_1) \times S(U_k U_1)) \\ G_2(\mathbb{C}^{2k+2}) = SU_{2k+2}/S(U_{2k} U_2) \\ E_6/Spin_{10} U_1 \end{cases}$$

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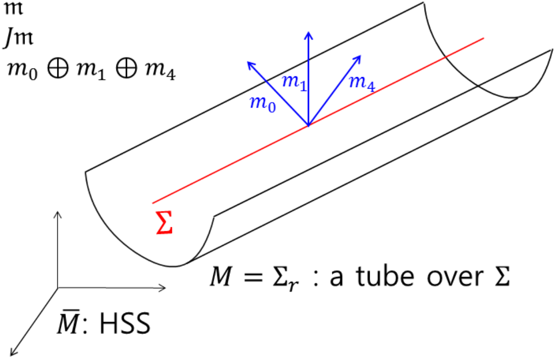
- $\Sigma = S^k,\ \mathbb{R}P^k, \mathbb{C}P^k, \mathbb{H}P^k, \mathbb{O}P^2$: RSS with rank 1
- Σ : totally real by the Kaehler structure J
- $T_0\bar{M} = T_0\Sigma \oplus \nu_0\Sigma$
- $T_0\Sigma \cong \mathfrak{m}$
- $\nu_0\Sigma \cong J\mathfrak{m}$
  $= m_0 \oplus m_1 \oplus m_4$



Principal curvatures

Now let us denote by M_t the tubes of radius t around Σ, with t sufficiently small. We choose $\frac{u_\delta}{|u_\delta|}$ for the direction of the normal geodesic in $\bar{M}$ with $\gamma(0) = o$ and $\dot{\gamma}(0) = \frac{u_\delta}{|u_\delta|}$.

We consider the End($\gamma^\perp$)-valued Jacobi differential equation

$$D'' + \bar{R}_\gamma^\perp \circ D = 0.$$

Then the shape operator $S(t)$ of M_t with respect to $-\dot{\gamma}(t)$ is given by

$$S(t) = D'(t) \circ D^{-1}(t).$$

For $Y(r) \in Jm_0 \subset T_0\Sigma$, and $\bar{M} \neq Q^k$. Then $\bar{R}^{\perp}_{z} \circ Y(t) = 4D(t)$ and

$$D'' + \bar{R}^{\perp}_{\gamma} \circ D = 0$$

gives

$$D(t) = (c_1 \cos(2t) + c_2 \sin(2t))Y(t),$$

with initial conditions $D(0) = c_1 Y(o)$ and $D'(0) = 2c_2 Y(0) = 0$ gives

$$D(t) = \cos(2t)Y(t).$$

So

$$S^{r}_{\dot{\gamma}(r)}(\cos(2r)Y(r)) = S^{r}_{\dot{\gamma}(r)}D(r) = -D'(r) = 2\sin(2r)Y(r)$$

implies

$$S^{r}_{\dot{\gamma}(r)}Y(r) = 2\tan(2r)Y(r).$$

Proposition 5.2

Let M_r be the tube of radius r around the totally geodesic totally real $\mathbf{R}P^k$ in $\mathbf{C}P^k$. Then

1. M_r is a Hopf hypersurface.
2. Principal curvature spaces and multiplicities are given

principal curvature	multiplicity	eigenspace
$2\tan(2r)$	1	Jm_0
$\tan(r)$	$k-1$	Jm_1
$-\cot(r)$	$k-1$	m_1

3. M_r is contact, $S\phi + \phi S = -2\cot(2r)\phi$.

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principal curvature	multiplicity	eigenspace
$2\tan(2r)$	1	Jm_0
$\tan(r)$	$2k-2$	Jm_1
0	1	Jm_4
$-\cot(r)$	$2k-2$	m_1
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Let M_r be the tube of radius $0 < t < \frac{\pi}{\sqrt{2}}$ around the totally geodesic totally real $\mathbf{H}P^k$ in $G_2(\mathbf{C}^{2k+2})$. Then

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1. M_r is a Hopf hypersurface.
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principal curvature	multiplicity	eigenspace
$-2\tan(2r)$	1	Jm_0
$\tan(r)$	8	Jm_1
0	7	Jm_4
$-\cot(r)$	8	m_1
$-2\cot(2r)$	7	m_4

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principal curvature	multiplicity	eigenspace
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$-\cot(r)$	$k-1$	m_1

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Converse of the Proof

1. If $\mathrm{rk}(\bar{M}) > 1$, what are the constraints on the unit normal field N ?
   If $\bar{M}$ is the complex quadric, then N is singular everywhere. Moreover, only one of possible two types of singular vectors can occur, namely that of $\mathfrak{A}$-principal vectors.
2. What are the constraints on the rank of $\bar{M}$?
3. Can we make a contradiction if $\mathrm{rk}(\bar{M}) > 2$?
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$\mathrm{rk}(\bar{M}) \leq 2$

Let Σ be an integrable and totally real submanifold of the holomorphic distribution $\mathcal{C}$ of M in $\bar{M}$. Since Σ is totally real in $\mathcal{C}$ such that $T_p\Sigma \oplus T_p^{\perp}\Sigma = \mathcal{C}$. Then for any $X, Y \in T_p\Sigma$

$$
\begin{aligned}
&[X, Y] \in T_p\Sigma \iff \\
&\quad 0 = g([X, Y], \xi) = -g((\phi A + A\phi)X, Y) \\
&\quad\quad = -kg(\phi X, Y).
\end{aligned}
$$

If $\dim T_p\Sigma \geq 2$, then $g(\phi X, Y) = 0$. This is in a contradiction to $Y = \phi X$. So $\mathrm{rank}\Sigma \leq 1$, which means that

$$1 \geq \mathrm{rank}\Sigma = \frac{1}{2}\mathrm{rank}\mathcal{C} = \frac{1}{2}\mathrm{rank}\bar{M}.$$

Consequently, $\mathrm{rank}\bar{M} \leq 2$.

Other related topics in Q^m

Let M be a real hypersurface in the complex quadric Q^m. Then the following problems related to the Ricci tensor are proved

- Parallel Ricci tensor $\nabla \mathrm{Ric} = 0$ (Adv. Math., Suh, 2015)
- Harmonic curvature $\delta \mathrm{Ric} = 0$, that is, $(\nabla_X \mathrm{Ric})Y = (\nabla_Y \mathrm{Ric})X$ (J. Math. Pures Appl., Suh, 2016)
- Pseudo-anti commuting Ricci tensor, that is, $\phi \mathrm{Ric} + \mathrm{Ric}\phi = k\phi$ and Ricci soliton problems (J. Math. Pures Appl., Suh, 2017)
- Pseudo-Einstein real hypersurfaces, $\mathrm{Ric} = ag + b\eta \otimes \xi$ (Math. Nachr., Suh, 2017)
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- Pseudo-Einstein real hypersurfaces, $\mathrm{Ric} = ag + b\eta \otimes \xi$ (Math. Nachr., Suh, 2017)
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Other related topics in Q^m

Let M be a real hypersurface in the complex quadric Q^m. Then the following problems related to the Ricci tensor are proved

- Parallel Ricci tensor $\nabla \mathrm{Ric} = 0$ (Adv. Math., Suh, 2015)
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Complex space form for $\alpha = g(S\xi, \xi)$

Let $M \subset (\bar{M}, \bar{g})$ be a Kaehler manifold. Then $\xi = -JN$ is said to be a *Reeb* vector field, and $\alpha = g(S\xi, \xi)$ a *Reeb* function, where S denotes the shape opeartor defined by $\bar{\nabla}_X N = -SX$ for any $X \in T_x M$, $x \in M$.

Theorem 3.6.

Let M be a real hypersurface in a complex space form $\bar{M}^n(c)$, $n > 2$. Then have the following:

- In $P_n(C)$, M is Hopf, then the Reeb function α is constant. (1976, Maeda, JMS)
- In $H_n(C)$, M is Hopf, then the Reeb function α is constant. (1990, Ki-Suh, OMJ)

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Compact Grassmannian for $\alpha = g(S\xi, \xi)$

Theorem 3.7.

Let M be a real hypersurface in Complex two-plane Grassmannian, $G_2(\mathbf{C}^{m+2})$, $n > 2$. Then the following results hold:

- M is Hopf and $g(S\mathfrak{D}, \mathfrak{D}^\perp) = 0$, then M is congruent to a tube over $G_2(\mathbf{C}^{m+1})$ or $\mathbf{H}P_n$, $m = 2n$. Moreover, α is constant. (1999, Berndt-Suh, Monat.)
- M has an isometric Reeb flow, then α is constant. (2002, Berndt-Suh, Monat.)
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Theorem 3.8.

Let M be a real hypersurface in non-compact complex two-plane Grassmannian, $SU_{2,m}/SU_2{\cdot}SU_m$, $n > 2$. Then the following results hold:

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- M has an isometric Reeb flow, then α is constant. (2013, Suh, AAM.)
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Complex Quadric for $\alpha = g(S\xi, \xi)$

Theorem 3.9.

Let M be a connected orientable real hypersurface in complex quadric, $Q^m = SO_{m+2}/SO_m{\cdot}SO_2$, $m > 2$. Then the following results hold:

- M is Hopf with $\mathfrak{A}$-isotropic, then the Reeb function α is constant. (2013, Berndt-Suh, IJM.)
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Theorem 3.10.

Let M be a real hypersurface in complex hyperbolic quadric, $SO_{m,2}/SO_m{\cdot}SO_2$, $m > 2$. Then the following results hold:

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- M is Hopf with $\mathfrak{A}$-principal, then the Reeb function α is constant. (2019, Suh-Perez-Woo, PMD.)
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Compact HSSM for $\alpha = g(S\xi, \xi)$

Theorem 3.11.

Let M be a connected orientable real hypersurface in Hermitian symmetric space of compact type, that is, $G_k(\mathbf{C}^{m+2})$, Q^m, Sp_m/U_m, SO_{2m}/U_m, $E_6/Spin_{10}\cdot U_1$, and $E_7/E_6\cdot U_1$. Then the following statements hold:

- M is Isometric Reeb Flow, then the Reeb function α is constant. (2019, Berndt-Suh, CCM.)
- M is Contact, $S\phi + \phi S = k\phi$, $k \neq 0$, then the Reeb function α is constant. (Berndt-Suh, In preprint.)

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Non-Compact HSSM for $\alpha = g(S\xi, \xi)$

Theorem 3.12.

Let M be a connected orientable real hypersurface in Hermitian symmetric space of non-compact type, that is, $G_k^*(\mathbf{C}^{m+2})$, Q^{m*}, $Sp_m(\mathbf{R})/U_m$, SO_{2m}^*/U_m, $E_6^{-14}/Spin_{10}\cdot U_1$, and $E_7^{-25}/E_6\cdot U_1$. Then the following statements hold:

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References I

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References II

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References III

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- Y.J. Suh, *Real hypersurfaces in the complex quadric with parallel Ricci tensor*, Advances in Mathematics, **281**(2015), 886-905.
- Y.J. Suh, *Real hypersurfaces in the complex quadric with harmonic curvature*, Journal de Math. Pures Appl., **106**(2016), 393–410.
- Y.J. Suh, *Pseudo-anti commuting and Ricci soliton real hypersurfaces in the complex quadric*, Journal de Math. Pures Appl., **107**(2017), 429-450.
- Y.J. Suh, *Real hypersurfaces in the complex hyperbolic quadric with isometric Reeb flow*, Comm. Contemp. Math. **20**(2018), 1750031(20 pages).

References III

- Y.J. Suh, *Real hypersurfaces in the complex quadric with parallel Ricci tensor*, Advances in Mathematics, **281**(2015), 886-905.
- Y.J. Suh, *Real hypersurfaces in the complex quadric with harmonic curvature*, Journal de Math. Pures Appl., **106**(2016), 393–410.
- Y.J. Suh, *Pseudo-anti commuting and Ricci soliton real hypersurfaces in the complex quadric*, Journal de Math. Pures Appl., **107**(2017), 429-450.
- Y.J. Suh, *Real hypersurfaces in the complex hyperbolic quadric with isometric Reeb flow*, Comm. Contemp. Math. **20**(2018), 1750031(20 pages).

References IV

- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians $SU_{2,m}/S(U_2{\cdot}U_m)$*, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in Hermitian symmetric spaces of compact type*, to appear in Comm. Contemp. Math. (2020), 1950039 (33 pages).

References IV

- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians $SU_{2,m}/S(U_2{\cdot}U_m)$*, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in Hermitian symmetric spaces of compact type*, to appear in Comm. Contemp. Math. (2020), 1950039 (33 pages).

References IV

- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians $SU_{2,m}/S(U_2 \cdot U_m)$*, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in Hermitian symmetric spaces of compact type*, to appear in Comm. Contemp. Math. (2020), 1950039 (33 pages).

References IV

- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians $SU_{2,m}/S(U_2 \cdot U_m)$*, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in Hermitian symmetric spaces of compact type*, to appear in Comm. Contemp. Math. (2020), 1950039 (33 pages).

References V

- J. Berndt and Y.J. Suh, *Contact real hypersurfaces in Hermitian symmetric spaces*, in preparation.
- J. Berndt, S. Console and C.E. Olmos, *Submanifolds and holonomy*, Second edition. Monographs and Research Notes in Math., Chapmann and Hall/CRC. **2016**,
- J. Berndt and C. Olmos, *Maximal totally geodesic submanifolds and index of symmetric spaces*, J. Diff. Geom., **104**(2016), 187-217.
- S. Klein and Y.J. Suh, *Contact real hypersurfaces in the complex hyperbolic quadric*, Annali di Matematica Pura ed Applicata **198**(2019), 1481–1494.

References V

- J. Berndt and Y.J. Suh, *Contact real hypersurfaces in Hermitian symmetric spaces*, in preparation.
- J. Berndt, S. Console and C.E. Olmos, *Submanifolds and holonomy*, Second edition. Monographs and Research Notes in Math., Chapmann and Hall/CRC. **2016**,
- J. Berndt and C. Olmos, *Maximal totally geodesic submanifolds and index of symmetric spaces*, J. Diff. Geom., **104**(2016), 187-217.
- S. Klein and Y.J. Suh, *Contact real hypersurfaces in the complex hyperbolic quadric*, Annali di Matematica Pura ed Applicata **198**(2019), 1481–1494.

References V

- J. Berndt and Y.J. Suh, *Contact real hypersurfaces in Hermitian symmetric spaces*, in preparation.
- J. Berndt, S. Console and C.E. Olmos, *Submanifolds and holonomy*, Second edition. Monographs and Research Notes in Math., Chapmann and Hall/CRC. **2016**,
- J. Berndt and C. Olmos, *Maximal totally geodesic submanifolds and index of symmetric spaces*, J. Diff. Geom., **104**(2016), 187-217.
- S. Klein and Y.J. Suh, *Contact real hypersurfaces in the complex hyperbolic quadric*, Annali di Matematica Pura ed Applicata **198**(2019), 1481–1494.

References V

- J. Berndt and Y.J. Suh, *Contact real hypersurfaces in Hermitian symmetric spaces*, in preparation.
- J. Berndt, S. Console and C.E. Olmos, *Submanifolds and holonomy*, Second edition. Monographs and Research Notes in Math., Chapmann and Hall/CRC. **2016**,
- J. Berndt and C. Olmos, *Maximal totally geodesic submanifolds and index of symmetric spaces*, J. Diff. Geom., **104**(2016), 187-217.
- S. Klein and Y.J. Suh, *Contact real hypersurfaces in the complex hyperbolic quadric*, Annali di Matematica Pura ed Applicata **198**(2019), 1481–1494.

References VI

- S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Math., **34**, American Math. Soc., 2001.
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Kobayashi and K. Nomizu, *Foundation of Differential Geometry*, Vol. II, A Wiley-Interscience Publ., Wiley Classics Library Ed., 1966.
- S.Kobayashi and T. Nagano, *On Filtered Lie Algebras and Geomteric Structures I*, J. of Math. and Mechanics, **13-5**(1964), 875-907.

References VI

- S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Math., **34**, American Math. Soc., 2001.
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Kobayashi and K. Nomizu, *Foundation of Differential Geometry*, Vol. II, A Wiley-Interscience Publ., Wiley Classics Library Ed., 1966.
- S.Kobayashi and T. Nagano, *On Filtered Lie Algebras and Geomteric Structures I*, J. of Math. and Mechanics, **13-5**(1964), 875-907.

References VI

- S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Math., **34**, American Math. Soc., 2001.
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Kobayashi and K. Nomizu, *Foundation of Differential Geometry*, Vol. II, A Wiley-Interscience Publ., Wiley Classics Library Ed., 1966.
- S.Kobayashi and T. Nagano, *On Filtered Lie Algebras and Geomteric Structures I*, J. of Math. and Mechanics, **13-5**(1964), 875-907.

References VI

- S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Math., **34**, American Math. Soc., 2001.
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Kobayashi and K. Nomizu, *Foundation of Differential Geometry*, Vol. II, A Wiley-Interscience Publ., Wiley Classics Library Ed., 1966.
- S.Kobayashi and T. Nagano, *On Filtered Lie Algebras and Geomteric Structures I*, J. of Math. and Mechanics, **13-5**(1964), 875-907.

ENDING

THANKS FOR *YOUR ATTENTION!*