Book of Abstracts

Symmetry and shape
Celebrating the 60th birthday of Prof. J. Berndt

28 - 31 October 2019

Santiago de Compostela, Spain

http://xtsunxet.usc.es/berndt2019/

Welcome

According to Felix Klein, geometry is the study of those properties in space that are invariant under a given transformation group. Intuitively, symmetry is the correspondence of shape at every point of a space. An interesting problem in geometry and many physical sciences is to determine the symmetries of a space from its shape. In Riemannian geometry, the natural group to consider is the isometry group, that is, the group of transformations of a manifold that preserve distances.

The aim of this conference is to gather experts from around the world in the study of symmetry in submanifold geometry, whilst we celebrate Jürgen Berndt's 60th birthday. The conference will revolve around the study of submanifolds of symmetric spaces, homogeneous submanifolds, including cohomogeneity one and polar actions, their characterization via concepts like isoparametric submanifolds or singular Riemannian foliations and their interaction with other topics in Differential Geometry and Geometric Analysis.

Contents

Organization

Organizing committee

José Carlos Díaz Ramos, Universidade de Santiago de Compostela, Spain.
Miguel Domínguez Vázquez, Universidade de Santiago de Compostela, Spain.
Eduardo García Río, Universidade de Santiago de Compostela, Spain.
Víctor Sanmartín López, Universidade de Santiago de Compostela, Spain.
Hiroshi Tamaru, Osaka City University, Japan.

Scientific committee

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Eduardo García Río, Universidade de Santiago de Compostela, Spain.
Claudio Gorodski, Universidade de São Paulo, Brazil.
Andreas Kollross, Universität Stuttgart, Germany.
Hiroshi Tamaru, Osaka City University, Japan.

Sponsors

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Schedule

The scientific programme is composed of plenary lectures, short talks of 25 min and of 15 min, and a poster session. For short talks of 15 min there will be two parallel sessions. All lectures will take place in Aula Magna of the Faculty of Mathematics, except for the second parallel session, which correspond to the second-named speaker for each slot in the schedule below, and whose lectures will be held in the Salón de Graos of the same building.

	Monday	Tuesday	Wednesday	Thursday
9:00	Documentation	A. Di Scala	A. Kollross	T. Murphy
9:30	Opening			
10:00	H. Tamaru	Y. J. Suh	A. Siffert	J. C. Díaz-Ramos
10:30				
11:00	Coffee break	Coffee break	Coffee break	Coffee break
11:30	L. Guijarro	T. Kajigaya	V. Palmer	A. Rodríguez
12:00	T. Hashinaga	V. Sanmartín	J. Van der Veken	S. Klein
12:30	L. Cavenaghi / S. Caeiro	M. Czarnecki / O. Pérez	M. Djoric / G. Kim	M. Moruz / A. Nicoli
12:50	N. Koike / M. Milijevic	A. Kubo / A. Seo	M. Ortega / H. Tadano	F. Manfio / R. Mariño
13:10	Lunch	Lunch	Lunch	Lunch
16:00	C. Gorodski	M. Alexandrino		C. Olmos
16:30				
17:00	Coffee break	Coffee break	Excursion + Social dinner	Closing
17:30	J. C. González-Dávila	M. Amann		
18:00	C. Woo / M. Zarei	D. González		
18:20	T. Jentsch / A. Wijffels			
18:30				
18:40				

Practical information

Internet connection

You can connect via **eduroam**, if your affiliation supports it and you have your devices correctly configured.

If not, you can connect to the wireless network **symmetry**. In this case, after establishing the connection, open your browser and in the identification page use the following data:

Login: **symmetry**
Password: **G9ZD%BUpW46+**

Restaurants

There are several options to have lunch near the conference venue:

- The cafeteria-canteens of the Faculty of Mathematics (same building as the conference), of the Escola Técnica Superior de Enxeñaría - ETSE (200m), or the cafeterias Fonseca and Rodríguez-Cadarso (300m) offer menus for less than 6 euros. Because of students' schedule, it is recommended to have lunch before 13:30.
- Some restaurants near the campus, such as *Santos*, *Xugo*, *Xantar* or *Altamira*, offer lunch menus for around 10-12 euros.
- Downtown and in the historic center there are many kinds of restaurants. Vegan and vegetarian options can be found, for example, at *Boca a boca*, *Malak Bistro*, *A Tulla* or *The Veggie Carmen*. Many restaurants offer typical Galician food, such as *O Dezaseis* or *María Castaña*.

Excursion and social dinner

On Wednesday 30 afternoon each participant will have the opportunity to join one of these two activities:

- A guided walking tour to visit the main squares, streets and charming spots of the historic city of Santiago de Compostela, of around 2 hours duration.
- Weather permitting, a hiking route along the Sarela river and up to the mount Pedroso, of around 12 kilometers length and 400 meters of cumulative elevation gain.

The social dinner of the conference will take place at Restaurante *Don Quijote* on **Wednesday at 20:30**. It will be free for all registered participants, but please **confirm your attendance upon registration** on Monday 28 morning, and choose main course (meat/fish/vegan).

Abstracts

Invited talks

On mean curvature flow of singular Riemannian foliations: non compact cases

Marcos Alexandrino
Universidade de São Paulo, Brazil
Aula Magna - Tuesday 29, 16:00

In this talk I will discuss the mean curvature flow (MCF) of a regular leaf of a closed generalized isoparametric foliation as initial datum, generalizing previous results of Marco Radeschi and of myself. As we will see, under bounded curvature conditions, any finite time singularity is a singular leaf, and the singularity is of type I.

I will also discuss the existence of basins of attraction, how cylinder structures can affect convergence of basic MCF of immersed submanifolds and will make a few remarks on MCF of non-closed leaves of generalized isoparametric foliation.

This talk will be based on a joint work with Leonardo Cavenaghi and Icaro Gonçalves (see https://arxiv.org/abs/1909.04201 preprint (2019)).

Homogeneous submanifolds in complex space forms

J. Carlos Díaz Ramos
Universidade de Santiago de Compostela, Spain
Aula Magna - Thursday 31, 10:00

This talk is a summary of the results obtained in collaboration (or under the influence of) Prof. J. Berndt. Although not exclusively, most of this research is focused on the study, classification and characterization of several classes of extrinsically homogeneous submanifolds in complex projective and hyperbolic spaces. These classes include orbits of polar actions, isoparametric hypersurfaces, hypersurfaces with constant principal curvatures and hypersurfaces with a small number of principal curvatures, among others.

The normal holonomy group of complex submanifolds

Antonio Di Scala
Politecnico di Torino, Italy
Aula Magna - Tuesday 29, 09:00

This talk is going to be a survey talk of results, ideas and questions about the normal holonomy group of complex submanifolds of complex space forms.

Actions on positively curved manifolds and boundary in the orbit space

Claudio Gorodski
Universidade de São Paulo, Brazil
Aula Magna - Monday 28, 16:00

We study isometric actions of compact connected Lie groups G on complete orientable positively curved n-manifolds M whose orbit space X has non-empty boundary in the sense of Alexandrov geometry, and prove that for sufficiently large n (in terms of G), there exists a positive-dimensional normal subgroup of G (possibly equal to G) that has a fix point in M and contains all isotropy groups associated to strata of codimension one of X. Among our applications, we classify representations of simple Lie groups whose orbit space has non- empty boundary (joint work with A. Kollross and B. Wilking).

Some recent results on polar actions

Andreas Kollross
Universität Stuttgart, Germany
Aula Magna - Wednesday 30, 09:00

Polar actions are special isometric Lie actions on Riemannian manifolds. An action is polar if there exists a section, i.e. a submanifold which intersects all orbits orthogonally. Polar actions are strongly and in many ways interconnected with the theory of Riemannian symmetric spaces. I will talk about about some recent results and open questions, in particular: classifications, infinitesimally polar actions, homogeneous spaces with polar isotropy and asystatic actions.

Complex Riemannian foliations of Kaehler manifolds

Thomas Murphy
Cal. State Fullerton, USA
Aula Magna - Friday 31, 09:00

For many natural problems arising in Riemannian geometry, the Kaehler setting is restrictive enough to allow concrete classification results. In this vein I will outline joint work with Paul-Andi Nagy classifying complex Riemannian foliations of any open subset of a Hermitian symmetric space of compact type. General results restricting such foliations on any Kaehler manifold are also derived. Time permitting we will discuss generalizations to other families of homogeneous spaces.

The index of symmetric spaces

Carlos Olmos
Universidad Nacional de Córdoba, Argentina
Aula Magna - Thursday 31, 16:00

We will speak about a joint project and work with *Jürgen Berndt* about the index of a symmetric space.

Being the classification of totally geodesic submanifolds of symmetric spaces hopeless, except for rank one or two, Onishchik defined in the eighties the concept of index of an irreducible symmetric space M: the minimal codimension of totally geodesic submanifold. It is a rather well-known result that the index of M is 1 (i.e. it admits a totally geodesic hypersurface) if and only if M has constant curvature. Onishchik himself determined the spaces with index two.

Our starting point, for dealing with the index, was to prove in 2014 that the index is always bounded from below by the rank of the symmetric space. Moreover, we conjectured that the index coincides with the reflective index except for the space G_2/SO_4 (the reflective index is the minimal codimension of a reflective totally geodesic submanifold that we computed from the work of Leung in the seventies).

In several articles, with essentially geometric tools, we were able to determine the index of almost all symmetric spaces, verifying the conjecture. In the case of exceptional symmetric spaces this was done in cooperation with *J. S. Rodríguez*. The conjecture remains open only for three classical families of symmetric spaces. Any of such families seems to require different methods and we have an strategy for each one. This, hopefully, would prove the conjecture and solve the index problem for all symmetric spaces.

Construction of harmonic mappings

Anna Siffert
Max-Planck-Institut, Germany
Aula Magna - Wednesday 30, 10:00

Geometric variational problems frequently lead to analytically extremely hard, nonlinear partial differential equations, where the standard methods fail. Thus finding nontrivial solutions is challenging. The idea is to study solutions with a certain minimum level of symmetry (i.e. group actions with low cohomogeneity), and use the symmetry to reduce the original problem to systems of non-linear ordinary differential equations, typically with singular boundary values. In my talk I explain how to construct harmonic mappings between manifolds with a lot of symmetry (i.e. cohomogeneity one manifolds). If time permits, I will discuss applications of the developed methods.

Real hypersurfaces in Hermitian symmetric spaces and related topics

Young Jin Suh
Kyungpook National University, South Korea
Aula Magna - Tuesday 29, 10:00

In this talk, first we introduce some back grounds and motivations for real hypersurfaces in Kaehler manifolds from 20th century. Morever, we give some new notions of isometric Reeb flow and contact hypersurfaces in Kaehler manifolds.

Next we investigate the structure of isometric Reeb flow and contact hypersurfaces in Hermitian symmetric spaces of compact type and of non-compact type, and give the complete classification of real hypersurfaces with isometric Reeb flow, and conjectures in Hermitian symmetric spaces. Moreover, we will show the other related topics and the constancy of the Reeb function.

Geometry of homogeneous hypersurfaces in noncompact symmetric spaces

Hiroshi Tamaru
Osaka City University, Japan
Aula Magna - Monday 28, 10:00

According to the results by Berndt-Brück and Berndt-Tamaru, cohomogeneity one actions on irreducible Riemannian symmetric spaces of noncompact type can be divided into the following three classes: (K) there is a unique singular orbit, (A) all orbits are regular and there is a unique minimal orbit, or (N) all orbits are isometrically congruent to each other. Note that regular orbits of these actions are precisely homogeneous hypersurfaces.

In this talk, we review several recent results on geometry of these homogeneous hypersurfaces, including the classification problems, intrinsic geometry, and extrinsic geometry.

Short talks (25 min)

On the topology of transitive and cohomogeneity one actions

Manuel Amann
University of Augsburg, Germany
Aula Magna - Tuesday 29, 17:30

Homogeneous spaces and manifolds of cohomogeneity one, i.e. manifolds admitting isometric actions with a one-dimensional orbit space, form rather important classes and, most importantly, rich sources of examples in Riemannian geometry. They are particularly of interest in the field of non-negative curvature metrics. Despite their significance, they still leave a lot of room for open questions.

In this talk I shall illustrate how on the one hand side we can answer classical questions from equivariant cohomology (such as "equivariant formality") for new manifold subclasses. On the other hand—motivated by the manifold counterpart and by the point of view of non-negative sectional curvature—we shall discuss questions related to the topology and equivariant cohomology in the singular framework of cohomogeneity one Alexandrov spaces. This talk reports on several joint projects in progress.

Invariant Ricci-flat Kähler metrics on tangent bundles of compact symmetric spaces

José Carmelo González-Dávila
Universidad de la Laguna, Spain
Aula Magna - Monday 28, 17:30

Let G/K be a compact Riemannian symmetric space of any rank and let $T(G/K)$ be its tangent bundle. From the natural G-equivariant diffeomorphism between $T(G/K)$ and the complexification $G^{\mathbb{C}}/K^{\mathbb{C}}$ of G/K, $T(G/K)$ can be endowed with a complex structure J_c^K, known as its *canonical complex structure*. We give a general description of all G-invariant Ricci-flat Kähler metrics on $T(G/K)$ associated to J_c^K. A detailed study of this technique can be found in arXiv: 1905.04308.

For it, we first describe all G-invariant Kähler structures $(\mathbf{g}, J_c^K)$ which are moreover Ricci-flat on an open dense subset $T^+(G/K)$ of $T(G/K)$. Here, $T^+(G/K)$ is the image of $G/H \times W^+$ under certain G-equivariant diffeomorphism, where W^+ is some Weyl chamber and H denotes the centralizer of a (regular) element of W^+ in K. Such G-invariant Kähler and Ricci-flat Kähler structures are determined univocally by a vector-function $\mathbf{a}\colon W^+ \to \mathfrak{g}_H$ satisfying certain conditions, $\mathfrak{g}_H$ being the subalgebra of $\mathrm{Ad}(H)$-fixed points of the Lie algebra of G.

Symmetry and non-negative curvature of vector bundles

David González
Universidad Politécnica de Madrid, Spain
Aula Magna - Tuesday 29, 18:00

In the first part of this talk we will discuss the existence of non-negatively curved metrics on equivariant vector bundles over homogeneous spaces (respectively over cohomogeneity one spaces with codimension two singular orbits). In the second part we will discuss which vector bundles over such spaces admit an equivariant structure.

Transverse Jacobi equation and soft theorems

Luis Guijarro
Universidad Autónoma de Madrid, Spain
Aula Magna - Monday 28, 11:30

We will review Wilking's transverse Jacobi equation (TJE), develop its comparison theory, and show how it can be used to extend several "classical" theorems to the realm of intermediate Ricci curvature (for instance, Berger's sphere theorem or Wilking's connectivity principle). Joint work with Fred Wilhelm.

Realizations of contact metric (κ, μ)-spaces as homogeneous real hypersurfaces in noncompact real two-plane Grassmannians

Takahiro Hashinaga
National Institute of Technology, Japan
Aula Magna - Monday 28, 12:00

(κ, μ) space is a class of contact metric manifolds which contains Sasakian manifolds and many non-Sasakian contact metric manifolds including standard examples of contact metric manifolds, such as the unit tangent sphere bundles of a Riemannian manifold with constant sectional curvature $c \neq 1$. In this talk, we consider realization problems of (κ, μ) spaces as real hypersurfaces in some specific Kähler manifolds. We will introduce that some of (κ, μ) spaces can be realized as homogeneous real hypersurfaces in the noncompact dual of real two-plane Grassmannians. This talk is based on joint work with J. T. Cho, A. Kubo, Y. Taketomi and H. Tamaru.

Some homogeneous Lagrangian submanifolds in complex hyperbolic spaces

Toru Kajigaya
Tokyo Denki University, Japan
Aula Magna - Tuesday 29, 11:30

We give some constructions and a partial classification result of homogeneous Lagrangian submanifolds in complex hyperbolic spaces. First, we show compact homogeneous Lagrangian submanifolds in $\mathbb{C}H^n$ completely correspond to the ones in $\mathbb{C}^n$ via a symplectic diffeomorphism. Next, we construct and classify non-compact homogeneous Lagrangian submanifolds in $\mathbb{C}H^n$ obtained by the solvable part of the Iwasawa decomposition. It is a joint work with Takahiro Hashinaga (NIT, Kitakyushu College).

Totally geodesic submanifolds in the Riemannian symmetric spaces of rank 2

Sebastian Klein
University of Mannheim, Germany
Aula Magna - Thursday 31, 12:00

The simplest type of submanifold in a Riemannian manifold are the totally geodesic submanifolds. While in a general Riemannian manifold there do not need to exist any totally geodesic submanifolds of dimension ≥ 1, it is known that any Riemannian symmetric space contains totally geodesic submanifolds of higher dimension.

The classification of all totally geodesic submanifolds in a Riemannian symemtric space is in general a complicated problem. It has been solved for the spaces of rank 1 by Wolf, and a partial solution was given for the spaces of rank 2 by Chen and Nagano. For spaces of rank ≥ 3, still no full classifications are known.

In my talk I will discuss the full classification of all totally geodesic submanifolds in all the irreducible Riemannian symmetric spaces of rank 2 (using different methods than the partial solution by Chen/Nagano). This is based on work done while I worked together with Professor Jürgen Berndt at the University College Cork in 2006-8. I am deeply grateful to him for the generous support and guidance he gave me during that time and thereafter.

Stochastically complete and proper solitons of MCF and IMCF confined in a ball

Vicente Palmer
Universitat Jaume I, Spain
Aula Magna - Wednesday 30, 11:30

We study some potential theoretic properties of homothetic solitons Σ^n of the MCF and the IMCF, and the influence of these properties on its geometry. Using the analysis of the extrinsic distance function defined on these submanifolds in $\mathbb{R}^{n+m}$, we observe similarities and differences among the solitons in both flows.

In particular, we study, by one hand, the geometric behavior of stochastically complete MCF and IMCF-solitons confined in a ball, and we shall see that stochastically complete λ-self-shrinkers of the MCF only can be confined in a R-ball $B_R^{n+m}(\vec{0})$ if $R \geq \sqrt{\frac{n}{\lambda}}$, where the quantity $\sqrt{\frac{n}{\lambda}}$ is the critical radius that makes the sphere $S^{m+n-1}_{\sqrt{\frac{n}{\lambda}}}$ a λ-self-shrinker for the MCF.

We are going to see too, in the spirit of the results in [3], that parabolic self-shrinkers for the MCF, confined in a ball of radius $\sqrt{\frac{n}{\lambda}}$ realizes as minimal submanifolds of the sphere $\mathbb{S}^{n+m-1}(\sqrt{\frac{n}{\lambda}})$.

On the other hand, we shall see results of this type for stochastically complete (and parabolic) self-expanders for the IMCF confined now in any R-ball $B_R^{n+m}(\vec{0})$.

Finally, we shall present a dual description of the behavior of a MCF- self-shrinker when we change the hypothesis of parabolicity for the assumption that it is properly immersed. The key idea is that a properly immersed self-shrinker cannot lie globally on one side of a λ-self-shrinker sphere $S^{n+m-1}(\sqrt{\frac{n}{\lambda}})$ unless it is a minimal immersion into this sphere.

The results of this talk are part of the prepint [1], (work in collaboration with V. Gimeno), in connection to some results in [2], (work in collaboration with A. Hurtado and C. Rosales).

References:

[1] V. Gimeno, V. Palmer and C. Rosales, Parabolicity, Brownian exit time and properness of solitons of the direct and inverse Mean Curvature Flow, Preprint, 2018

[2] A. Hurtado, V. Palmer and C. Rosales, Parabolicity criteria and characterization results for submanifolds of bounded mean curvature in model manifolds with weights, Preprint, 2018

[3] S. Pigola and M. Rimoldi, Complete self-shinkers confined into some regions of the space, *Ann. Glob. Anal. Geom.* **45** (2014), 47-65.

Homogeneous and inhomogeneous isoparametric hypersurfaces in quaternionic hyperbolic spaces

Alberto Rodríguez-Vázquez
Universidade de Santiago de Compostela, Spain
Aula Magna - Thursday 31, 11:30

The problem of classifying cohomogeneity one actions in quaternionic hyperbolic spaces has been open for almost twenty years. Moreover, a solution for it would yield the classification of these actions on symmetric spaces of rank one.

In 2001, Berndt and Brück found a method to construct cohomogeneity one actions on noncompact symmetric spaces of rank one. Later, Berndt and Tamaru classified these actions on the complex hyperbolic space and on the Cayley hyperbolic plane.

In this talk, I will present the classification of cohomogeneity one actions on quaternionic hyperbolic spaces. As a very surprising by-product of our proof we find an uncountable number of examples of inhomogeneous isoparametric families of hypersurfaces with constant principal curvatures. To our knowledge, these isoparametric families constitute the only such examples known in Riemannian manifolds, apart from the celebrated Ferus, Karcher and Münzner hypersurfaces in spheres and an example in the Cayley hyperbolic plane.

Homogeneous submanifolds in symmetric spaces of non-compact type

Víctor Sanmartín-López
Universidade de Santiago de Compostela, Spain
Aula Magna - Tuesday 29, 12:00

In this talk we will address the study of certain homogeneous submanifolds in the setting of symmetric spaces of non-compact type. More precisely, we will construct, describe and classify under certain conditions CPC submanifolds (submanifolds whose principal curvatures, counted with multiplicities, do not depend on the normal directions) and austere submanifolds in symmetric spaces of non-compact type. In order to do so, we will make an extensive use of the algebraic information encoded in the root system of symmetric spaces of non-compact type.

Lagrangian submanifolds of the complex quadric

Joeri Van der Veken
KU Leuven, Belgium
Aula Magna - Wednesday 30, 12:00

The complex quadric Q^n is the complex hypersurface of complex $(n+1)$-dimensional projective space given in homogeneous coordinates by the equation $z_0^2+z_1^2+\ldots+z_{n+1}^2=0$. This manifold inherits a Kähler structure from the complex projective space, carries a family of non-integrable almost product structures and its curvature can be relatively easily described in terms of these two. Moreover, Q^n is the natural target space when considering the Gauss map of a hypersurface of a round sphere. In fact, such Gauss maps are related to minimal Lagrangian submanifolds of Q^n. We will discuss this relation – in particular for isoparametric hypersurfaces of spheres – and then study minimal Lagrangian submanifolds of Q^n, obtaining examples and some classifications, such as that of minimal Lagrangian submanifolds of Q^n with constant sectional curvature.

Short talks (15 min)

New examples of critical metrics for quadratic functionals

Sandro Caeiro
Universidade de Santiago de Compostela, Spain
Salón de Graos - Monday 28, 12:30

The purpose of this communication is to present new examples of critical metrics for quadratic curvature functionals by focusing on homogeneous metrics and locally conformally flat ones.

In dimension three (and more generally for any locally conformally flat manifold) all quadratic curvature functionals reduce to either $\mathcal{S}$ or $\mathcal{F}_t$ given by

$$\mathcal{S} : g \mapsto \int_M \tau^2 dvol_g, \qquad \mathcal{F}_t : g \mapsto \int_M \{\|\rho\|^2 + t\tau^2\} dvol_g,$$

Any Einstein metric is critical for $\mathcal{S}$ and $\mathcal{F}_t$ for any $t \in \mathbb{R}$. Our purpose is to show the existence of many non-Einstein critical metrics and to point out some of their properties. In particular non-Einstein examples which are critical for all quadratic functionals are given.

Positive Ricci curvature through Cheeger deformations

Leonardo Francisco Cavenaghi
University of São Paulo, Brazil
Aula Magna - Monday 28, 12:30

Let (M, g) be a compact Riemannian manifold with an isometric G-action. If a principal orbit has finite fundamental group and the quotient space has a metric with positive Ricci curvature, then Searle and Wilhem showed that M carries a G-invariant metric of positive Ricci curvature. To do so, they make use of a conformal change on the original metric and Cheeger deformations. The question remained whether Cheeger deformations are sufficient to prove the Theorem. In this short talk (or poster) we discuss how to approach this question studying the tensors associated to Cheeger deformations on singular points of the action. We both construct examples where the Cheeger deformation is not enough and give sufficient (algebraic) conditions where the deformation is enough. Once understood the geometry near singular points, we give a simplified proof for Searle-Wilhelm result.

This is a joint work with Renato J.M. e Silva and Llohann D. Sperança.

Bisectors and foliations in the complex hyperbolic space

Maciej Czarnecki
Uniwersytet Łódzki, Poland
Aula Magna - Tuesday 29, 12:30

In the complex hyperbolic space we consider metric properties of bisectors i.e. equidistants from pair of points. It is known that bisectors are in one-to-one correspondence

with pairs of points of the ideal boundary (vertices of the bisector's spine). Using boundary invariants like complex cross-ratio and the Goldman invariant we estimate distance between bisectors and discuss the problem of separating bisectors including relation with their spines. Bisector foliations give (through their slices) examples of totally geodesic codimension 2 foliations of the complex hyperbolic space.

Geometry of CR submanifolds

Mirjana Djoric
University of Belgrade, Serbia
Aula Magna - Wednesday 30, 12:30

It is a natural question to investigate submanifolds of (almost) complex Hermitian manifolds $(\overline{M}, \overline{g}, J)$ which have a special behavior with respect to the almost complex structure J. For that reason, the first classes of submanifolds investigated were the almost complex submanifolds (in which case J maps the tangent space into the tangent space) and totally real submanifolds (in which case J maps the tangent space in the normal space).

A natural generalization of the above classes of submanifolds are the so-called CR-submanifolds. A submanifold M of $\overline{M}$ is called a CR submanifold if there exist distributions $\mathcal{H}$ and $\mathcal{H}^{\perp}$ of constant dimension such that $\mathcal{H} \oplus \mathcal{H}^{\perp} = TM$, $J\mathcal{H} = \mathcal{H}$, $J\mathcal{H}^{\perp} \subset T^{\perp}M$. Note that totally real submanifolds $(\mathcal{H} = \{0\})$ and almost complex submanifolds $(\mathcal{H} = TM)$ are trivial examples of CR-submanifolds. Moreover, real hypersurfaces of almost Hermitian manifolds are typical examples of CR submanifolds of maximal CR dimension. In M. Djoric, M. Okumura, *CR submanifolds of complex projective space*, Developments in Mathematics **19**, Springer, 2009, we introduced the reader to the study of CR submanifolds of complex manifolds, especially of CR submanifolds of maximal CR dimension of complex projective space and we presented the original results on this topic by the authors. On this occasion, we present some new results.

We have also studied CR submanifolds of the nearly Kähler six sphere, see for example M. Djoric, L. Vrancken, Three dimensional minimal CR submanifolds in S^6 satisfying Chen's equality, *J. Geom. Phys.*, Volume 56, Issue 11, (2006), 2279-2288, and now we show some of our achievements.

Furthermore, the odd-dimensional analogue of CR-submanifolds in Kählerian manifolds is the concept of contact CR-submanifolds in Sasakian manifolds. Namely, a submanifold M in the Sasakian manifold $(\widetilde{M}, \varphi, \xi, \eta, \tilde{g})$ carrying a φ-invariant distribution $\mathcal{D}$, such that the orthogonal complement of $\mathcal{D}$ in TM is φ-anti-invariant, is called a *contact CR-submanifold*. Here we recall our study of four dimensional contact CR-submanifolds in $\mathbb{S}^5(1)$ and in $\mathbb{S}^7(1)$, see M. Djoric, M.I. Munteanu, L. Vrancken, Four-dimensional contact CR-submanifolds in $S^7(1)$, *Math. Nachr.*, 290 (16) (2017), 2585-2596 and we present some new results about the five-dimensional contact CR-submanifolds in $\mathbb{S}^7(1)$.

Jacobi relations on naturally reductive spaces

Tillmann Jentsch
University of Stuttgart, Germany
Aula Magna - Monday 28, 18:20

Naturally reductive spaces in general can be seen as an adequate generalization of symmetric spaces. Nevertheless there are non-symmetric naturally reductive spaces whose geometric properties come closer to symmetric spaces than others. We consider a distinguished class of non-symmetric naturally reductive spaces intimately related to special geometries in dimension six and seven. For this class we establish the following property: along every geodesic the Jacobi operator satisfies an ordinary differential equation with constant coefficients which can be choosen independent of the given geodesic.

Real hypersurfaces in complex hyperbolic quadric with Ricci soliton and pseudo-Einstein condition

Gyu Jong Kim
Woosuk University, South Korea
Salón de Graos - Wednesday 30, 12:30

As a kind of Hermitian symmetric space, complex hyperbolic quadric is the non-compact dual of the complex quadric. We present the differences between those two spaces and introduce the curvature tensor explicitly. In order to classify real hypersurfaces in Hermitian symmetric space, we also give a almost contact metric structure induced from the ambient space. Using these concepts, we give classification theorems of real hypersurfaces in complex hyperbolic quadric with Ricci soliton and pseudo-Einstein condition.

Mean curvature flow in symmetric space of compact type and regularized mean curvature flow in Hilbert space

Naoyuki Koike
Tokyo University of Science, Japan
Aula Magna - Monday 28, 12:50

In this talk, I first state the relation between the mean curvature flow in a symmetric space of compact type and the regularized mean curvature flow in a Hilbert space. In particular, I state the research of the mean curvature flow starting from equifocal submanifold by using the regularized mean curvature flow. Next I state applications of the research of the regularized mean curvature flow to the Gauge theory. In particular, I state a result for the concentration of the holonomy elements along the regularized mean curvature flow starting from a certain kind of regularizable hypersurface in the space of connections of a G-bundle, where G is a compact semi-simple Lie group. Also, I state that new examples of infinite dimensional isoparametric submanifold in a Hilbert space are constructed as submanifolds in the Hilbert space of H^0-connections of the G-bundle.

Cohomogeneity two actions on the complex hyperbolic plane

Akira Kubo
Hiroshima University, Japan
Aula Magna - Tuesday 29, 12:50

In this talk, we give a classification of cohomogeneity two actions on the complex hyperbolic plane, up to orbit equivalence: they are four polar actions, one non-polar action, and an uncountable family of non-polar actions.

Complete submanifolds with codimension two in Euclidean space

Fernando Manfio
University of São Paulo, Brazil
Aula Magna - Thursday 31, 12:50

In this talk we will present a complete classification of complete submanifolds in Euclidean space with cohomogeneity one and codimension two.

Weakly Einstein hypersurfaces in spaces of constant curvature

Rodrigo Mariño Villar
Universidade de Santiago de Compostela, Spain
Salón de Graos - Thursday 31, 12:50

Weakly Einstein metrics appear naturally in the study of critical metrics for determined functionals. Moreover, in dimension four, a universal identity given by Berger determines that if a metric is Einstein, then it is weakly Einstein, but the opposite does not hold in general, so it is a natural question to study non-Einstein metrics that satisfy these conditions. The aim of this talk is to classify 4-dimensional hypersurfaces in spaces of constant curvature with this special property.

Left invariant structures on statistical manifolds

Mirjana Milijevic
University of Banja Luka, Bosnia and Herzegovina
Salón de Graos - Monday 28, 12:50

On statistical manifolds there exists a connection different than the Levi Civita one. We give conditions for a statistical connection in terms of adjoint representation.

On product minimal Lagrangian submanifolds in complex space forms

Marilena Moruz
KU Leuven, Belgium
Aula Magna - Thursday 31, 12:30

In this paper we consider minimal Lagrangian submanifolds in n-dimensional complex space forms. More precisely, we study such submanifolds which, endowed with the induced metric, write as a Riemannian product of two Riemannian manifolds, each having constant sectional curvature. As the main result, we give a complete classification of these submanifolds.

A topological lower bound for the energy of a unit vector field on closed Euclidean hypersurfaces

Adriana Nicoli
University of São Paulo, Brazil
Salón de Graos - Thursday 31, 12:30

For a unit vector field on a closed immersed Euclidean hypersurface M^{2n+1}, $n \geq 1$, we exhibit a nontrivial lower bound for its energy which depends on the degree of the Gauss map of the immersion. Two non-homotopic immersions will possess two different normal degrees; the bigger this value, the bigger the energy of a given unit vector field. When the hypersurface is the unit sphere $\mathbb{S}^{2n+1}$, this lower bound corresponds to a well established value from the literature. We introduce a list of functionals $\mathcal{B}_k$ on a compact Riemannian manifold M^m, $1 \leq k \leq m$, and show that, when the underlying manifold is a closed hypersurface, these functionals possess similar properties regarding the degree of the immersion. In addition, Hopf flows minimize $\mathcal{B}_n$ on $\mathbb{S}^{2n+1}$.

Hopf real hypersurfaces in the indefinite complex projective space

Miguel Ortega
Universidad de Granada, Spain
Aula Magna - Wednesday 30, 12:50

H. Anciaux and K. Panagiotidou obtained the very basic results for real hypersurfaces in the indefinite complex projective space $\mathbb{C}P^n_p$ of index $1 \leq p \leq n-1$. (See [1]).

We wish to further develop their ideas. We re-obtain the almost contact metric structure on a real hypersurface in $\mathbb{C}P^n_p$, namely (g, ϕ, ξ, η). We construct new examples, which we call *real hypersurfaces of type* A_+, A_-, B_0, B_+, B_- *and* C, because they are somehow similar to those in the famous Takagi's and Montiel's lists. All our examples are Hopf. We also exhibit an example of a lightlike Hopf real hypersurface. We prove that two real hypersurfaces with the same shape operator are linked by a holomorphic isometry of the ambient space. Next, we classify the non-degenerate real hypersurfaces in indefinite complex projective space such that $AX = \lambda X + \rho\eta(X)\xi$, where λ and ρ are smooth functions. Finally, we classify those non-degenerate real hypersurfaces such that $A\phi = \phi A$.

This talk is based on the paper [2].

References:

[1] H. Anciaux, K. Panagiotidou, Hopf Hypersurfaces in pseudo-Riemannian complex and para-complex space forms, *Diff. Geom. Appl.* **42** (2015) 1-14.

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Homogeneous CR submanifolds in complex hyperbolic spaces

Olga Pérez-Barral
Universidade de Santiago de Compostela, Spain
Salón de Graos - Tuesday 29, 12:30

A submanifold of a Hermitian manifold is said to be CR (or Cauchy-Riemann) if each tangent space can be decomposed into an orthogonal direct sum of a totally real space and a complex space. The aim of this talk is to study homogeneous CR submanifolds in complex hyperbolic spaces.

Weakly 1-completeness of holomorphic fiber bundles over compact Kähler manifolds

Aeryeong Seo
Kyungpook National University, South Korea
Salón de Graos - Tuesday 29, 12:50

In 1985 Diederich and Ohsawa proved that every disc bundle over a compact Kähler manifold is weakly 1-complete. In this talk, under certain conditions I will talk about a generalization of this result to the case of fiber bundles over compact Kähler manifolds whose fibers are classical bounded symmetric domains. Moreover if the bundle is obtained by the diagonal action on the product of irreducible bounded symmetric domains of classical type, it is hyperconvex.

Some New Myers-Type Theorems via m-Bakry-Émery Ricci Curvature

Homare Tadano
Tokyo University of Science, Japan
Salón de Graos - Wednesday 30, 12:50

We establish some new compactness criteria for complete Riemannian manifolds assuming some asymptotic conditions on the m-Bakry-Émery Ricci curvature. Our results generalize Myers-type theorems for complete Riemannian manifolds via m-Bakry-Émery Ricci curvature obtained by M. Fernández-López and E. García-Río (Math. Ann. 340 (2008), 893-896), M. Limoncu (Arch. Math. (Basel) 95 (2010), 191-199, Math. Z. 271 (2012), 715-722), Z. Qian (Quart. J. Math. Oxford Ser. (2) 48 (1997), 235-242), G. Wei and W. Wylie (J. Differential Geom. 83 (2009), 377-405), and J.-Y. Wu (Ann.

Global Anal. Geom. 54 (2018), 541-549). The key ingredients in proving our results are the Bochner formula via m-Bakry-Émery Ricci curvature and the Laplacian comparison theorem for the Witten-Laplacian.

Lagrangian submanifolds of the complex hyperbolic quadric

Anne Wijffels
KU Leuven, Belgium
Salón de Graos - Monday 28, 18:20

The complex hyperbolic quadric Q^{*n} is the complex hypersurface of complex anti-de Sitter space CH_1^{n+1}, given in homogeneous coordinates by the equation $-z_0^2 - z_1^2 + \cdots + z_n^2 + 1 = 0$.

This manifold inherits a Kähler structure from the complex anti-de Sitter space and the shape operators define almost product structures on Q^{*n}. Its curvature can relatively easy be described in terms of these two structures. Moreover, the Grassmannian Q^{*n} is the natural target space when considering the Gauss map of a spacelike hypersurface of anti-de Sitter space $H_1^{n+1}(1)$ In fact, such Gauss maps are related to Lagrangian submanifolds of Q^{*n}. Therefore we are particularly interested in the Lagrangian immersions in the complex hyperbolic quadric.

The maximal existence condition of Ricci parallel Hopf real hypersurfaces in complex Grassmannians of rank two

Changhwa Woo
Pukyong National University, South Korea
Aula Magna - Monday 28, 18:00

In this talk, we introduce a notion of parallel Ricci tensor for real hypersurfaces in complex Grassmannians of rank two. We try to find the maximal subbundle which guarantees the existence of real hypersurfaces in the given ambient spaces.

Equivariant classification of cohomogeneity one Alexandrov spaces in low dimensions

Masoumeh Zarei
University of Augsburg, Germany
Salón de Graos - Monday 28, 18:00

Alexandrov spaces are a synthetic generalization of Riemannian manifolds with a lower curvature bound. It is then a natural question that to what extent one can generalize the basic results of the Riemannian manifolds with a lower curvature bound to Alexandrov spaces. In this talk, I will explore this question in the context of cohomogeneity one actions on Alexandrov spaces and I will give an equivariant classification of such actions in low dimensions. Furthermore, I will discuss a characterization of cohomogeneity one Alexandrov spaces to be topological manifolds and show that in low dimensions such spaces are homeomorphic to smooth manifolds. This is a joint work with Fernando Galaz-Garcia.

Posters

Studying shape through derived categories

Beatriz Álvarez Díaz
Universidade de Santiago de Compostela, Spain

In the 1960s, Grothendieck developed a general theory of duality. The appropriate context to achieve this goal is the theory of derived categories. To construct the derived category of an abelian category, one inverts the morphisms of complexes that induce isomorphisms in cohomology, obtaining a category where objects with the same cohomology are identified up to isomorphism. As it is known, cohomology allows us to study the shape of the objects we are working with, and derived categories provide a very efficient tool to deal with cohomology.

Characterization of simply-connected homogeneous spaces with geometric structures

José Luis Carmona Jiménez
Universidad Complutense de Madrid, Spain

Simply-connected and complete homogeneous Riemannian spaces are characterized by the famous result of Ambrose and Singer as those spaces admitting a (1,2)-tensor S such that $\tilde{\nabla}S = 0$, $\tilde{\nabla}R = 0$ and $\tilde{\nabla}g = 0$, for $\tilde{\nabla} = \nabla^{LC} + S$. The result was extended to pseudo-Riemannian manifolds as well as pseudo-Riemannian manifolds endowed with addition geometric structures defined by tensors (as Kähler, quaternion-Kähler, Sasaki just to mention some cases).

The goal of this poster is the presentation of similar results in the case where the manifold is not necessarily pseudo-Riemannian. A main instance is the study of homogenous symplectic manifolds. We also analyze the local and global approach.

Loxodromic unit vector field on punctured spheres

Jackeline Conrado
University of São Paulo, Brazil

Let M be a closed oriented Riemannian manifold and $\vec{v}\colon M \to T^1M$ a unit vector field on M, where T^1M is equipped with the Sasaki metric. The volume of $\vec{v}$ is defined on [6] and [7]. When M is an antipodally punctured sphere, a relation between the volume and the Poincaré index of a vector field was established in [3]. It follows from the proof that the north-south unit vector field realize this lower bound. A natural question arises: Is this the only one with this property? The answer is no. We prove that the loxodromic unit vector fields are unique with this property. Precisely: *The lower bound for the volume is realized if, and only if, $\vec{v}$ is a loxodromic unit vector field*.

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On inaudible properties with constant scalar curvature

José Manuel Fernández-Barroso
Universidad de Extremadura, Spain

The inverse spectral geometry looks for properties which one can determine from the eigenvalues of the Laplace operator acting on functions. Following M. Kac, we say that such properties are audible. Many properties related with the curvature operator has been proved to be inaudible as to be D'Atri space or to be a type $\mathcal{A}$ manifold. On the other hand, a typical example of audible properties are the classical heat invariants. Here, we prove the inaudibility of being a $\mathcal{C} \setminus \{\mathcal{A} \cup \mathcal{B}\}$ space, in the sense of A. Gray.

In this way, we also prove that one can not determine by the spectrum of the Laplace operator if a Riemannian manifold M is k-D'Atri for each k, $k = 3, \ldots, \dim M - 1$.

Anti-self-dual para-Kähler surfaces with constant scalar curvature

María Ferreiro Subrido
Universidade de Santiago de Compostela, Spain

In this work we give a description of the local structure of anti-self-dual para-Kähler surfaces with constant scalar curvature. Taking into account the fact that the underlying structure of para-Kähler manifolds is that of a Walker manifold, we show that any anti-self-dual para-Kähler surface is locally isometric to the cotangent bundle of a flat affine surface equipped with a modified Riemannian extension given by a $(1,1)-$tensor field T on the affine surface.

Dynamics on finite posets

David Mosquera Lois
Universidade de Santiago de Compostela, Spain

We generalize both the concept of Morse function on posets introduced by Minian [1] to Lyapunov function and his result on integration of Morse matchings to general matchings. Moreover, we present results on differentiation of Morse and Lyapunov functions, which can be seen as converses or duals to the integration results.

References:

[1] E. G. Minian: Some remarks on Morse theory for posets, homological Morse theory and finite manifolds, *Topology Appl.* **159**.12 (2012), 2860-2869.

HNN-extensions of Lie superalgebras

Pilar Páez-Guillán
Universidade de Santiago de Compostela, Spain

The Higman-Neumann-Neumann extensions (HNN-extensions) of groups were introduced in 1949 to prove that every countable group can be embedded into a group with two generators. Since then, the idea of the construction has been extended in several ways to other algebraic structures such as semigroups, rings, Lie algebras or Leibniz algebras, allowing to introduce new embedding theorems in each case.

In this work, we construct HNN-extensions of Lie superalgebras and prove that every Lie superalgebra embeds into any of its HNN-extensions. As an application, we show that any Lie superalgebra with countable dimension embeds into a two-generated Lie superalgebra.

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